Section 2.5 – Division of Fractions and Applications

Objective a: Reciprocal of a Fraction.

Two numbers are called **Reciprocals** if their product is 1. To find the reciprocal of a non-zero fraction, switch the numerator and denominator.

Find the reciprocal of the following:

Ex. 1a $\frac{2}{3}$ Ex. 1b $\frac{17}{6}$

Ex. 1c 8 Ex. 1d 0

Solution:

a) Switching the numerator and denominator, we get: $\frac{3}{2}$

b) Switching the numerator and denominator, we get: $\frac{6}{17}$

c) Since 8 = $\frac{8}{1}$, then the reciprocal is $\frac{1}{8}$.

d) Since $\frac{1}{0}$ is undefined, then 0 has no reciprocal.

Find the reciprocal of a fraction is sometimes called inverting.

Objective b: Division of Fractions.

In general, division of fractions is too difficult to do directly. For a problem like: $\frac{6}{5} \div \frac{4}{7}$, we are asking how many times $\frac{4}{7}$ goes into $\frac{6}{5}$? This is very difficult. To see how we can work in this problem, let's examine a division problem for which we know the answer.

Ex. 2 6 donuts ÷ 2 people

Solution:

We all know that 6 ÷ 2 = 3 donuts per person. Another way to think of this problem is that each person gets *half* of the six donuts.

So, $6 \div 2 = \frac{6}{1} \div \frac{2}{1} = \frac{6}{1} \bullet \frac{1}{2} = \frac{\cancel{2} \cdot 3}{1} \bullet \frac{1}{\cancel{2} \cdot 1} = \frac{3}{1} = 3$.

Notice that dividing by 2 is the same thing as multiplying by 1/2. Thus, in order to divide two fractions, we change the operation to multiplication and invert ("flip") the fraction to the right. Then, we proceed as before with multiplying fractions.

"flip"

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$$

Change to multiplication

Note the "reciprocal" of $\frac{c}{d}$ is $\frac{d}{c}$.

Simplify. Write your answers as mixed numbers if possible.

Ex. 3 $\frac{6}{5} \div \frac{4}{7}$

Solution:

Invert the fraction on the right and multiply: $\frac{6}{5} \div \frac{4}{7} = \frac{6}{5} \bullet \frac{7}{4} =$

$\frac{\cancel{2}\cdot 3}{5} \bullet \frac{7}{\cancel{2}\cdot 2} = \frac{21}{10} = 2\frac{1}{10}$.

Ex. 4 $\frac{3}{14} \div \frac{7}{8}$

Solution:

It may be tempting to reduce the seven and fourteen, but reducing only works in multiplication, not in division (division as we saw in chapter one is not commutative). We must invert and multiply:

$\frac{3}{14} \div \frac{7}{8} = \frac{3}{14} \bullet \frac{8}{7} = \frac{3}{\cancel{2}\cdot 7} \bullet \frac{\cancel{2}\cdot 4}{7} = \frac{12}{49}$.

Ex. 5 $\frac{5}{6} \bullet \frac{9}{10}$

Solution:

The operation is already multiplication; do not invert the fraction:

$\frac{5}{6} \bullet \frac{9}{10} = \frac{1\cdot \cancel{5}}{2\cdot \cancel{3}} \bullet \frac{\cancel{3}\cdot 3}{2\cdot \cancel{5}} = \frac{3}{4}$.

Ex. 6 $0 \div \frac{3}{4}$

Solution:

Since 0 divided by any non-zero number is zero, then $0 \div \frac{3}{4} = 0$

Ex. 7 $\frac{9}{10} \div 0$

Solution:

Since division by zero is undefined, then $\frac{9}{10} \div 0$ is undefined.

Let's now explore more closely way division by zero is undefined by looking at a concrete example.

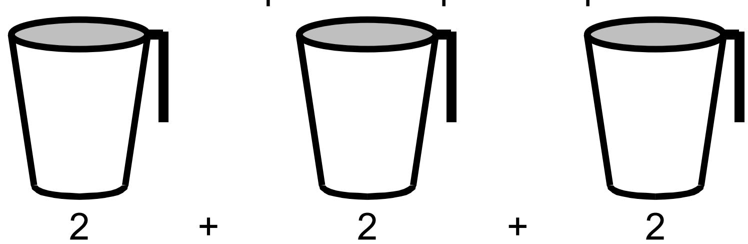
Solve the following:

Ex. 8 To make three batches of chocolate chip cookies, Mara needs six cups of flour. Find how many scoops of flour will she need if her scoop holds:

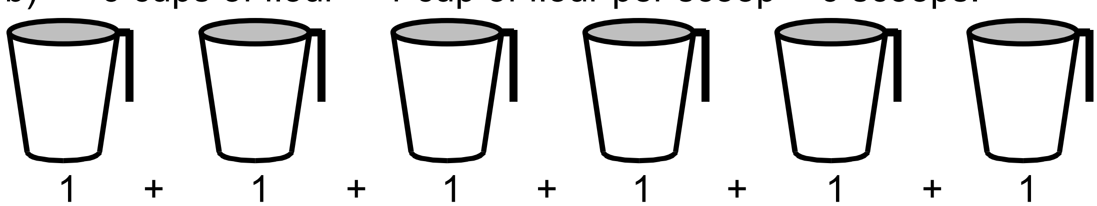
a) 2 cups of flour.
b) 1 cup of flour.
c) $\frac{1}{2}$ cup of flour.
d) $\frac{1}{4}$ cup of flour.
e) $\frac{1}{8}$ cup of flour.
f) 0 cups of flour.

Solution:

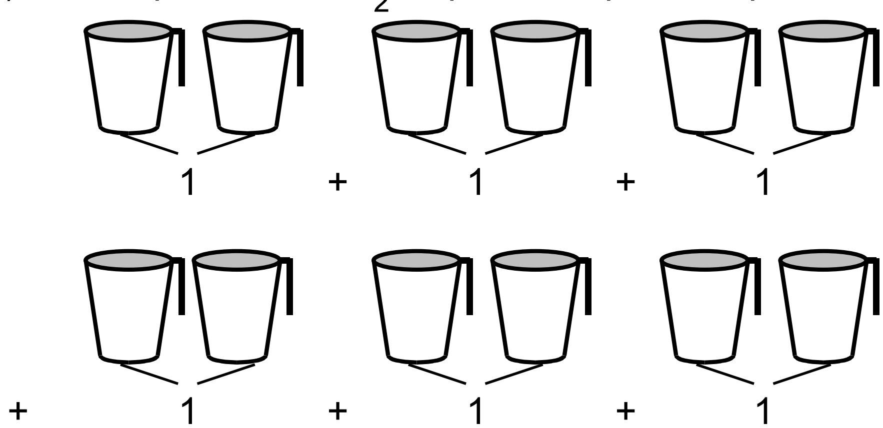
a) 6 cups of flour ÷ 2 cups of flour per scoop = 3 scoops.



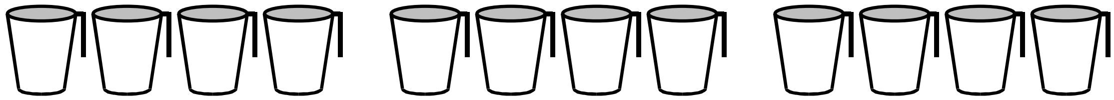
b) 6 cups of flour ÷ 1 cup of flour per scoop = 6 scoops.



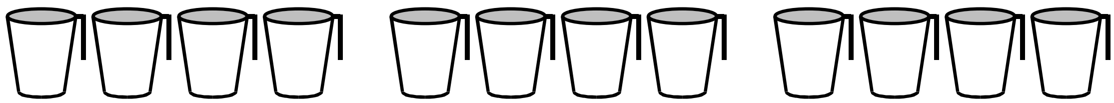
c) 6 cups of flour ÷ $\frac{1}{2}$ cup of flour per scoop = 6•2 = 12 scoops.



d) 6 cups of flour ÷ $\frac{1}{4}$ cup of flour per scoop = 6•4 = 24 scoops.

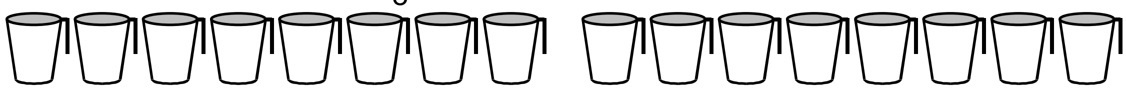


1 + 1 + 1

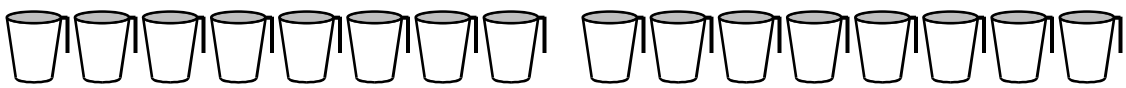


+ 1 + 1 + 1

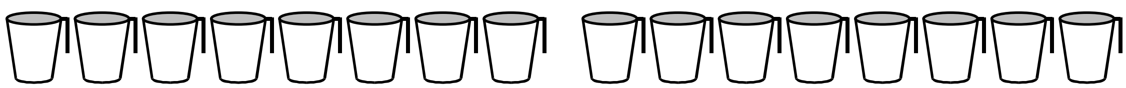
e) 6 cups of flour ÷ $\frac{1}{8}$ cup of flour per scoop = 48 scoops.



1 + 1



+ 1 + 1



+ 1 + 1

Notice the trend, the smaller the scoop size, the more scoops are needed to get 6 cups of flour. This is because the smaller scoops hold less and thus it takes more scoops to complete the six cups.

f) To solve the problem 6 cups of flour ÷ 0 cups of flour per scoop, suppose the scoop had no bottom. It cannot hold any flour so no matter how many scoops Mara takes; she will never be able to get one cup let alone six cups of flour. She can continue scooping forever and we will never end up with any flour in her mixing bowl. So, she cannot make her cookies using this scoop. Thus,
6 cups of flour ÷ 0 cups of flour per scoop = Cannot be done. This means that 6 ÷ 0 is undefined.

Note, for 0 ÷ 2, think of needing zero cups of milk; Mara would use her 2-cup scoop zero times since she needs no milk. Thus, 0 ÷ 2 = 0.

Objective c: Order of Operations.

Simplify:

Ex. 9 $\frac{8}{15} \div \frac{24}{35} \div \frac{2}{3}$

Solution:

Invert and multiply from left to right:

$\frac{8}{15} \div \frac{24}{35} \div \frac{2}{3}$ (Invert $\frac{24}{35}$ and change the first ÷ to •)

$= \frac{8}{15} \bullet \frac{35}{24} \div \frac{2}{3}$ (Reduce the first two fractions)

$= \frac{1 \bullet \cancel{8}}{3 \bullet \cancel{5}} \bullet \frac{7 \bullet \cancel{5}}{3 \bullet \cancel{8}} \div \frac{2}{3}$

$= \frac{1}{3} \bullet \frac{7}{3} \div \frac{2}{3}$ (Invert $\frac{2}{3}$ and change the last ÷ to •)

$= \frac{1}{3} \bullet \frac{7}{3} \bullet \frac{3}{2}$ (Reduce)

$= \frac{1}{3} \bullet \frac{7}{1 \bullet \cancel{3}} \bullet \frac{\cancel{3} \bullet 1}{2}$

$= \frac{7}{6}$ or $1\frac{1}{6}$.

Ex. 10 $\left(\frac{5}{7} \div \frac{2}{35}\right)^2 \div 125$

Solution:

$\left(\frac{5}{7} \div \frac{2}{35}\right)^2 \div 125$ (Inside (), invert $\frac{2}{35}$ and change the first ÷ to •)

$= \left(\frac{5}{7} \bullet \frac{35}{2}\right)^2 \div 125$ (Reduce inside of ())

$= \left(\frac{5}{7 \bullet 1} \bullet \frac{7 \bullet 5}{2}\right)^2 \div 125$

$= \left(\frac{5}{1} \bullet \frac{5}{2}\right)^2 \div 125$ (Multiply inside ())

$= \left(\frac{25}{2}\right)^2 \div 125$ (Exponents)

$= \left(\frac{25}{2} \bullet \frac{25}{2}\right) \div 125$

$= \left(\frac{625}{4}\right) \div 125$ (Write 125 over 1)

$= \frac{625}{4} \div \frac{125}{1}$ (Invert and multiply)

$= \frac{625}{4} \bullet \frac{1}{125}$ (Reduce)

$= \frac{5 \bullet 125}{4} \bullet \frac{1}{1 \bullet 125} = \frac{5}{4} \bullet \frac{1}{1} = \frac{5}{4}$ or $1\frac{1}{4}$

Objective d: Applications.

Ex. 11 How many sections of pipe each $\frac{5}{6}$ feet long can be cut from a pipe that is 15 feet long?

Solution:

We need to take the total length and divide it by the length of each piece: $15 \div \frac{5}{6} = \frac{15}{1} \div \frac{5}{6} = \frac{15}{1} \bullet \frac{6}{5} = \frac{3 \bullet 5}{1} \bullet \frac{6}{1 \bullet 5} = 18$

The plumber will be able to cut 18 pieces of pipe.

Ex. 12 Juan is considering relocating from his $36,000 a year job in Austin, Texas to a similar job in Oakland, CA. If he must have $\frac{8}{5}$ of his current salary in CA to maintain his current standard of living, what will his salary need to be in CA?

Solution:

Since he must have $\frac{8}{5}$ of his salary, we need to multiply $\frac{8}{5}$ with $36,000: $\frac{8}{5} \bullet 36000 = \frac{8}{5} \bullet \frac{36000}{1} = \frac{8}{1 \bullet 5} \bullet \frac{7200 \bullet 5}{1} = 57{,}600$

His salary will need to be $57,600.

Ex. 13 A Developer has $\frac{129}{8}$ acres of land she plans to sell in lots of $\frac{3}{16}$ acres. How many lots will she sell?

Solution:

We need to take the total acreage divide by the lot size:

$$\frac{129}{8} \div \frac{3}{16} = \frac{129}{8} \bullet \frac{16}{3} = \frac{3 \bullet 43}{1 \bullet 8} \bullet \frac{2 \bullet 8}{3 \bullet 1} = 86$$

She will be able to sell 86 lots.

Ex. 14 After an accident, a damaged car has lost $\frac{3}{4}$ worth of its retail value. If its retail value was $3200 prior to the accident,

a) How much did the car lose in value?

b) What is the car worth now?

Solution:

a) We need to find $\frac{3}{4}$ of $3200: $\frac{3}{4} \bullet 3200 = \frac{3}{4} \bullet \frac{3200}{1} = \frac{3}{1 \bullet 4} \bullet \frac{800 \bullet 4}{1}$ $= 2400$ The car lost $2400 in value.

b) $3200 – $2400 = $800

The car is now worth $800 in value.