Sect 2.5 - Applications Involving Percents

Concept #1 - Solving Basic Percent Equations.

The word percent means "out of 100" or "per 100". We use the symbol "%" to denote a percent. Thus, we can write fourteen percent as 14%.
Some examples are:

Percent	Interpretation
1) 87% of the students receive financial aid	87 out of every 100 students receive financial aid.
2) Sales tax rate is 8%	$8 in sales tax is charged for every $100 purchased.
3) 6% commission rate.	$6 in commission is earned for every $100 in sales.

Many applications involving percents can be solved using the basic percent equation: The amount is a percent of the base.
The verb "to be" acts as equals and the word "of" indicates the operation of multiplication. Thus, we can translate this sentence into an equation:

Basic Percent Equation:

The sentence: "The amount is a percent of the base." translates into

amount = (percent)(base)

where the percent is converted into a fraction or decimal for computation.

Solve the following:

Ex. 1 What is 73% of 950?

Solution:

The base is 950, and the percent is 73%. Plugging into our sentence, we get:

The amount is 73% of 950.

Let a = the amount

a is 73% of 950. (translate)

a = (73%)(950) (convert the percent to a decimal)
a = 0.73(950) (multiply)
a = 693.5

Thus, the amount is 693.5.

Ex. 2 -342 is -19% of what?

Solution:

The amount is – 342 and the percent is – 19%. Plugging into our sentence, we get:

$\underline{-342}$ is $\underline{-19\%}$ of the $\underline{\text{base}}$.

Let b = the base.

$\underline{-342}$ is $\underline{-19\%}$ of $\underline{b}$. (translate)

$-342 = (-19\%)b$ (convert the percent to a decimal)

$-342 = -0.19b$

$\frac{-342}{-0.19} = \frac{-0.19b}{-0.19}$ (divide both sides by – 0.19)

$1800 = b$

So, the base is 1800.

Ex. 3 What percent of 85 is – 59.5?

Solution:

The amount is – 59.5 and the base is 85. Plugging into our sentence, we get:

$\underline{-59.5}$ is a $\underline{\text{percent}}$ of $\underline{85}$.

Let p = the percent

$\underline{-59.5}$ is $\underline{p}$ of $\underline{85}$. (translate)

$-59.5 = p(85)$

$\frac{-59.5}{85} = \frac{85p}{85}$ (divide both sides by 85)

$-0.7 = p$ (convert to a percent)

$-70\% = p$

Hence, the percent is – 70%.

The key to solving applications with percents is distilling the problem down into a simple sentence like the sentences we solved in examples #1 - #3. We will start by identifying the amount, the percent, and the base and filling in the sentence: $\underline{\text{ Amount }}$ is a $\underline{\text{ Percent }}$ of the $\underline{\text{ Base }}$

Afterwards, we will set-up the percent equation and solve the problem.

Ex. 4 A basketball auditorium increased its 24,000 seating capacity by 15%. How many seats were added to the auditorium?

Solution:
We first fill in our simple sentence:

Seats Added is 15% of 24000.

So, in this problem, the base is 24000, the percent is 15%, and we are looking for the amount:

Let a = the amount

a is 15% of the 24000. (translate)

a = (15%)(24000) (convert to a decimal)

a = 0.15(24000) (multiply)

a = 3600

So, 3600 seats were added.

Ex. 5 A computer system that sold for $2,700 one year ago can now be bought for $1,161. What percent change does this represent?

Solution:
We first need to compute the change: $1161 – $2700 = – $1539
Now, we fill in our simple sentence:

– $1539 is a Percent of $2700.

The amount is – 1539, the base is 2700, so we are looking for the percent:

Let p = the percent

– 1539 is p of 2700. (translate)

– 1539 = p(2700)

$$\frac{-1539}{2700} = \frac{2700p}{2700}$$ (divide both sides by 2700)

– 0.57 = p (convert to a percent)

– 57% = p

Hence, the change percent is – 57% or a 57% decrease.

Ex. 6 Richard left a tip of $5.40 for a meal at The Flying Tomato Pizzeria. If this was 15% of his total bill, what was his total bill (to the nearest cent)?

Solution:
We first fill in our sentence:

$5.40 is 15% of the total bill.

The amount is $5.40, the percent is 15%, so we are looking for the base:

Let b = the base.

5.4 is 15% of b. (translate)

5.4 = (15%)b (convert the percent to a decimal)
5.4 = 0.15b

$\frac{5.4}{0.15} = \frac{0.15b}{0.15}$ (divide both sides by 0.15)
36 = b
So, the total bill was $36.

Concept #2 Sales Tax and Commission

Sales tax is the sales tax rate times the price of the merchandise
or Sales tax = (tax rate)(price)

Commission is the commission rate times the total sales.
or Commission = (commission rate)(sales)

Ex. 7 Juan purchased a High-Definition TV priced at $749.95. If the sales tax rate was 8.25%, what was the total amount he paid?
Solution:
We need to find the sales tax and then add that to the price.
We first fill in our sentence:
Sales Tax is 8.25% of $749.95.
So, in this problem, the base is 749.95, the percent is 8.25%, and we are looking for the amount:
Let a = the amount
a is 8.25% of the $749.95. (translate)

a = (8.25%)(749.95) (convert to a decimal)
a = 0.0825(749.95) (multiply)
a = 61.870875 ≈ $61.87
The total amount paid is the price plus tax:
Total Paid = $749.95 + $61.87 = $811.82
Thus, Juan paid a total of $811.82.

Ex. 8 With sales tax, a cordless phone system costs $102.72. What was the price before sales tax if the sales tax rate is 8.125%.
Solution:
The total amount paid is the price plus sales tax:
$102.72 = price + sales tax

But, Sales tax is 8.125% of price:

Let p = the price before sales tax.

Sales tax is 8.125% of p: (translate)

Sales tax = (8.125%)p (convert to a percent)

Sales tax = 0.08125p

and \$102.72 = price + sales tax

\$102.72 = p + sales tax

In the equation \$102.72 = price + sales tax, replace sales tax by ***0.08125p***:

\$102.72 = p + ***sales tax***

102.72 = p + ***0.08125p*** (combine like terms)

102.72 = 1.08125p

$\frac{102.72}{1.08125} = \frac{1.08125p}{1.08125}$ (divide both sides by 1.08125)

95.00115... = p

p ≈ \$95

The price before sales tax was \$95.

Ex. 9 If Joe received \$4200 in commission on the sale of a \$90,000 home, what was his commission rate?

Solution:

\$4200 is a percent of \$90,000

Let p = the percent

4200 is p of 90000. (translate)

4200 = p(90000)

$\frac{4200}{90000} = \frac{90000p}{90000}$ (divide both sides by 90000 & reduce)

$\frac{14}{300} = p$ (convert to a percent)

$\frac{14}{300} \bullet \frac{100\%}{1} = p$ (reduce)

$p = \frac{14}{3} \bullet \frac{1\%}{1}$ (multiply)

$p = \frac{14}{3}\% = 4\frac{2}{3}\%$

Hence, Joe's commission rate was $4\frac{2}{3}\%$.

Concept #3 Simple Interest

When a person gets a loan from the bank, that person has to pay back the amount of the loan plus interest. The interest is the fee that the person has to pay for using the money from the bank. In the same way, if a person opens a savings account, the bank pays that person interest since the bank gets to use that person's money while it is in the account.
The formula for simple interest is:

$$I = prt$$

The Interest, **I**, is the fee paid for borrowing money. The amount of the loan or the amount in a savings account is called the principal, **p**. The interest rate, **r**, is the rate that interest is charged on the amount borrowed converted to a decimal or fraction. The interest rate is usually given as a percentage and is assumed to be a yearly percentage unless it is states otherwise. The amount of time, **t**, is the length of time of the loan in years. In instances where the interest rate is given as monthly interest rate or a daily interest rate, then correspondingly, the time will have to be in months or in days. The amount, **A**, that is paid back is the principal plus interest (**A = p + i**).

Many financial institutions do not like the fact that the number of days in a month is inconsistent. Some months have 31 days, some have 30, some have 28 and occasionally, there is a month that has 29 days. To avoid these inconsistencies, banks will assume that there are 30 days in a month. As a consequence, they will then assume that there are 12•30 = 360 days in a year. This is known as "Banker's Rule":

Banker's Rule:
1 month = 30 days
1 year = 12 months = 360 days

As you can imagine, there is potential for great confusion between financial institutions and the consumer since each has a different definition of what a month is. To avoid confusion, many times these institutions will state the length of time in days rather than in months. For example, many stores will offer "90 days same as cash" instead of "three months same as cash" since three months to financial institution is not the same as three months for a consumer. By offering it in days, everyone understands the exact length being discussed. Now, let's try some examples:

Ex. 10 Max borrowed \$4,500 at 6% simple interest for 2 years. What was the interest he had to pay and the total he had to pay back?

<u>Solution:</u>

In this problem, $p = \$4,500$, $r = 6\% = 0.06$, and $t = 2$ years. Plugging into $i = prt$, we get:

$$i = (\$4500)(0.06)(2) = \$540$$

The amount to be paid back, A, is principal plus interest which is $A = \$4500 + \$540 = \$5040$.

So, the interest is \$540 and the amount he had to pay back was \$5040.

Ex. 11 Alisa had a loan of \$600 at 7.8% simple interest for 120 days. How much did she have to pay back?

<u>Solution:</u>

In this problem, $p = \$600$, $r = 7.8\% = 0.078$, and $t = 120$ days. Before we plug into the formula, we will need to convert the days into years. Using Banker's Rule,

$$t = 120 \text{ days} = (120 \text{ days}) / (360 \text{ days}) = \frac{1}{3} \text{ years.}$$

Plugging into $i = prt$, we get:

$$i = (\$600)(0.078)(\tfrac{1}{3}) = \$15.60$$

So, the interest is \$15.60. The amount to be paid back, A, is principal plus interest which is $A = \$600 + \$15.60 = \$615.60$. Thus, she had to pay back \$615.60.

Ex. 12 Alejandro borrowed \$3600 for nine months. If he paid back \$3863.25, what was the simple interest rate?

<u>Solution:</u>

This problem is different from the first two in that we are given the amount he paid back and we need to solve for the interest rate. Since $p = \$3600$ and $A = \$3863.25$, we can use $A = p + i$ to find the interest:

$$\$3863.25 = \$3600 + i \quad \text{(subtract \$3600 from both sides)}$$
$$\underline{-\$3600 = -\$3600}$$
$$\$263.25 = i,$$

So, the interest, i, was \$263.25.

Since $t = 9$ months = (9 months) / (12 months) = 0.75 years and $p = \$3600$, we can plug into $i = prt$ to solve for r:

$$i = prt$$
$$\$263.25 = (\$3600)r(0.75 \text{ years})$$

$263.25 = ($3600)r(0.75 years) (multiply 3600 and 0.75)
263.25 = 2700r (divide both sides by 2700)

$$\frac{263.25}{2700} = \frac{2700r}{2700}$$

0.0975 = r or r = 0.0975 = 9.75%

The interest rate is 9.75%.

Ex. 13 Samuel has to repay $6,809.60 for the principal he borrowed at 7.2% for 3 years. How much was the principal?

Solution:

At first glance, it does seem that this problem can be solve since we do not the principal or the interest. What we do know is the total amount A. Since A = p + i, then $6,809.60 = p + i. Our first equation that we need to use is then:

EQN#1: $6,809.60 = p + i

We also know that i = prt. Plugging in r = 7.2% = 0.072 and t = 3 years, we get:

i = p(0.072)(3 years) (multiply 0.072 by 3)
i = 0.216p, so EQN#2 is **i = 0.216p.**

Going back to EQN#1, we can use EQN#2 to replace i by 0.216p:

$6,809.60 = p + **i** (replace i by 0.216p)
$6,809.60 = p + **0.216p** (combine like terms)
6809.60 = 1.216p (divide both sides by 1.216)

$$\frac{6809.60}{1.216} = \frac{1.216p}{1.216}$$

5600 = p or p = $5600

The principal was $5600.