

Sect 2.6 - Multiplying and Dividing Mixed Numbers

Objective a: Multiplying Mixed Numbers.

Multiplying mixed numbers presents a problem. When we say $5\frac{1}{4}$, this means $5 + \frac{1}{4}$. In multiplication, we cannot mix addition with multiplication.

To avoid this problem, we will change our mixed numbers into improper fractions and then multiply. To estimate mixed numbers, round each mixed number to the nearest whole number and perform the operation.

Simplify the following and then estimate the answer:

Ex. 1 $7\frac{7}{8} \times 3\frac{5}{9}$

Solution:

First, we change each mixed number to an improper fraction:

$$7\frac{7}{8} = \frac{7 \cdot 8 + 7}{8} = \frac{63}{8} \text{ and } 3\frac{5}{9} = \frac{3 \cdot 9 + 5}{9} = \frac{32}{9}.$$

$$\text{So, } 7\frac{7}{8} \times 3\frac{5}{9} = \frac{63}{8} \cdot \frac{32}{9} = \frac{9 \cdot 7}{1 \cdot 8} \cdot \frac{4 \cdot 8}{1 \cdot 9} = \frac{28}{1} = 28.$$

$$\text{Estimate: } 7\frac{7}{8} \times 3\frac{5}{9} \approx 8 \cdot 4 = 32$$

Ex. 2 $6\frac{3}{4} \cdot 4\frac{1}{5}$

Solution:

First, we change each mixed number to an improper fraction:

$$6\frac{3}{4} = \frac{6 \cdot 4 + 3}{4} = \frac{27}{4} \text{ and } 4\frac{1}{5} = \frac{4 \cdot 5 + 1}{5} = \frac{21}{5}$$

$$\text{So, } 6\frac{3}{4} \cdot 4\frac{1}{5} = \frac{27}{4} \cdot \frac{21}{5} = \frac{567}{20}. \text{ Now, let's write } \frac{567}{20} \text{ as a mixed number:}$$

$$\begin{array}{r} 28 \\ 20 \overline{) 567} \\ \underline{- 560} \\ 7 \end{array}$$

$$\text{Thus, } \frac{567}{20} = 28\frac{7}{20}.$$

$$\text{Estimate: } 6\frac{3}{4} \cdot 4\frac{1}{5} \approx 7 \cdot 4 = 28$$

Simplify the following:

Ex. 3 $8 \cdot 2\frac{5}{12}$

Solution:

First, we change each number to an improper fraction:

$$8 = \frac{8}{1} \text{ and } 2\frac{5}{12} = \frac{12 \cdot 2 + 5}{12} = \frac{29}{12}.$$

So, $8 \cdot 2\frac{5}{12} = \frac{8}{1} \cdot \frac{29}{12} = \frac{4 \cdot 2}{1} \cdot \frac{29}{4 \cdot 3} = \frac{58}{3}$. Now, let's write $\frac{58}{3}$ as a mixed number:

$$\begin{array}{r} 19 \\ 3 \overline{)58} \\ \underline{-57} \\ 1 \end{array}$$

Thus, $\frac{58}{3} = 19\frac{1}{3}$

Estimate: $8 \cdot 2\frac{5}{12} \approx 8 \cdot 2 = 16$

Ex. 4 $(3\frac{4}{5})(2\frac{1}{7})(0)(\frac{5}{9})(1\frac{1}{6})$

Solution:

Since the product of any number and zero is zero, then

$$(3\frac{4}{5})(2\frac{1}{7})(0)(\frac{5}{9})(1\frac{1}{6}) = 0.$$

Objective b: Dividing Mixed Numbers.

Just like in multiplication, we need to change any whole numbers and mixed numbers into improper fractions before proceeding.

Simplify the following:

Ex. 5 $56 \div \frac{7}{8}$

Solution:

Rewrite 56 as an improper fraction, invert 7/8 and multiply:

$$56 \div \frac{7}{8} = \frac{56}{1} \div \frac{7}{8} = \frac{56}{1} \cdot \frac{8}{7} = \frac{7 \cdot 8}{1} \cdot \frac{8}{7 \cdot 1} = \frac{64}{1} = 64.$$

Ex. 6 $\frac{7}{2} \div 8$

Solution:

Rewrite 8 as an improper fraction, invert 8/1 and multiply:

$$\frac{7}{2} \div 8 = \frac{7}{2} \div \frac{8}{1} = \frac{7}{2} \cdot \frac{1}{8} = \frac{7}{16}.$$

Simplify the following and then estimate the answer:

Ex. 7 $6\frac{3}{8} \div 3\frac{7}{9}$

Solution:

First, we change each number to an improper fraction:

$$6\frac{3}{8} = \frac{6 \cdot 8 + 3}{8} = \frac{51}{8} \text{ and } 3\frac{7}{9} = \frac{3 \cdot 9 + 7}{9} = \frac{34}{9}.$$

$$\text{So, } 6\frac{3}{8} \div 3\frac{7}{9} = \frac{51}{8} \div \frac{34}{9} = \frac{51}{8} \cdot \frac{9}{34} = \frac{3 \cdot 17}{8} \cdot \frac{9}{2 \cdot 17} = \frac{27}{16}$$

Now, write the answer as mixed number:

$$\text{Thus, } \frac{27}{16} = 1\frac{11}{16}.$$

$$\text{Estimate: } 6\frac{3}{8} \div 3\frac{7}{9} \approx 6 \div 4 = \frac{6}{4} = \frac{3}{2} \text{ or } 1\frac{1}{2}.$$

Ex. 8 $\frac{6\frac{3}{4}}{7\frac{1}{2}}$

Solution:

This is still a division problem: $\frac{a}{b} = a \div b$ so the problem becomes:

$$\frac{6\frac{3}{4}}{7\frac{1}{2}} = 6\frac{3}{4} \div 7\frac{1}{2} = \frac{27}{4} \div \frac{15}{2} = \frac{27}{4} \cdot \frac{2}{15} = \frac{3 \cdot 9}{2 \cdot 2} \cdot \frac{2 \cdot 1}{3 \cdot 5} = \frac{9}{10}.$$

$$\text{Estimate: } 6\frac{3}{4} \div 7\frac{1}{2} \approx 7 \div 8 = \frac{7}{8}.$$

Objective c: Applications

Solve the following:

Ex. 9 A steak contains $2\frac{2}{3}$ servings per pound. How many pounds of steak are need for 176 servings?

Solution:

We will need to divide the total number of servings by the amount of servings per pound:

$$176 \div 2\frac{2}{3} = \frac{176}{1} \div \frac{8}{3} = \frac{176}{1} \cdot \frac{3}{8} = \frac{8 \cdot 22}{1} \cdot \frac{3}{1 \cdot 8} = 66$$

They will need 66 pounds of steak.

Ex. 10 A wedding cake recipe calls for $1\frac{3}{4}$ cups of sugar. If the recipe makes enough cake for 30 people, how many cups of sugar will be needed to make enough cake for 150 people?

Solution:

Take the total number of people and divided by the number of people one cake recipe can feed:

$$150 \div 30 = 5 \text{ cake recipes needed.}$$

Now take the amount of sugar needed for each cake recipe and multiply by the number of cake recipes needed:

$$5 \cdot 1\frac{3}{4} = \frac{5}{1} \cdot \frac{7}{4} = \frac{35}{4}.$$

But $\begin{array}{r} 8 \\ 4 \overline{)35} \\ \underline{-32} \\ 3 \end{array}$

So, $8\frac{3}{4}$ cups of sugar are needed.

Ex. 11 On a certain car, one turn of the tie-rod sleeve changes the toe-in setting by $\frac{3}{16}$ in. How many turns are required to change the toe-in setting by $1\frac{1}{4}$ in?

Solution:

We need to take the total change in the toe-in setting and divided by the change per turn:

$$1\frac{1}{4} \div \frac{3}{16} = \frac{5}{4} \div \frac{3}{16} = \frac{5}{4} \cdot \frac{16}{3} = \frac{5}{4 \cdot 1} \cdot \frac{4 \cdot 4}{3} = \frac{20}{3} = 6\frac{2}{3}$$

But $\begin{array}{r} 6 \\ 3 \overline{)20} \\ \underline{-18} \\ 2 \end{array}$

Hence, $6\frac{2}{3}$ turns are required.