Sect 3.3 - Adding and Subtracting Unlike Fractions

Objective a: Adding and Subtracting Unlike Fractions

Unlike fractions are fractions that have different denominators. We saw at the end of the last section that in order to add or subtract fractions with different denominators, we first find the L.C.M of the denominators which we will now call it the L.C.D., the **L**east **C**ommon **D**enominator. After finding the L.C.D., we build each fraction so it has a denominator equal to the L.C.D. Then we proceed as we did in Section 3.1.

Simplify the following:

Ex. 1 $\frac{9}{14} + \frac{1}{3} - \frac{5}{21}$

Solution:

We begin by finding the L.C.D. of 14, 3, and 21:
14 = 2•7, 3 = 3, and 21 = 3•7, so the L.C.D. = 2•3•7 = 6•7 = 42.
Next, we build the fractions so that they have a denominator equal to 42. Since 42 ÷ 14 = 3, times the top and bottom of the first fraction by 3. Since 42 ÷ 3 = 14, multiply the top and bottom of the second fraction by 14. Since 42 ÷ 21 = 2, times the top and bottom of the third fraction by 2.

$$\frac{9}{14}\frac{\bullet 3}{\bullet 3} + \frac{1}{3}\frac{\bullet 14}{\bullet 14} - \frac{5}{21}\frac{\bullet 2}{\bullet 2} = \frac{27}{42} + \frac{14}{42} - \frac{10}{42}$$

Now combine the numerators:

$$\frac{27}{42} + \frac{14}{42} - \frac{10}{42} = \frac{31}{42}.$$

Ex. 2 $\frac{8}{9} - \frac{5}{12} + \frac{4}{15}$

Solution:

We begin by finding the L.C.D. of 9, 12, and 15:
$9 = 3^2$, 12 = 4•3 = 2^2•3, and 15 = 3•5. So, the L.C.D. = 2^2•3^2•5 = 4•9•5 = 36•5 = 180. Since 180 ÷ 9 = 20, 180 ÷ 12 = 15, and 180 ÷ 15 = 12, multiply top and bottom of each fraction by the appropriate number and then combine:

$$\frac{8}{9}\frac{\bullet 20}{\bullet 20} - \frac{5}{12}\frac{\bullet 15}{\bullet 15} + \frac{4}{15}\frac{\bullet 12}{\bullet 12} = \frac{160}{180} - \frac{75}{180} + \frac{48}{180} = \frac{133}{180}.$$

Ex. 3 $\frac{7}{20} + \frac{11}{30} - \frac{2}{15}$

Solution:

$20 = 4 \bullet 5 = 2^2 \bullet 5$, $30 = 6 \bullet 5 = 2 \bullet 3 \bullet 5$, and $15 = 3 \bullet 5$. Thus, L.C.D. = $2^2 \bullet 3 \bullet 5 = 60$. Since $60 \div 20 = 3$, $60 \div 30 = 2$, and $60 \div 15 = 4$, then

$$\frac{7}{20} + \frac{11}{30} - \frac{2}{15} = \frac{7 \bullet 3}{20 \bullet 3} + \frac{11 \bullet 2}{30 \bullet 2} - \frac{2 \bullet 4}{15 \bullet 4} = \frac{21}{60} + \frac{22}{60} - \frac{8}{60} = \frac{35}{60}$$

But $\frac{35}{60}$ can be reduced: $\frac{35}{60} = \frac{5 \bullet 7}{5 \bullet 12} = \frac{7}{12}$.

Ex. 4 $\frac{1}{4} - \frac{1}{6} + \frac{5}{6} - \frac{11}{12}$

Solution:

$4 = 2^2$, $6 = 2 \bullet 3$, and $12 = 2^2 \bullet 3$. The L.C.D. = $2^2 \bullet 3 = 12$. Since the last fraction already has the L.C.D. as its denominator, it will stay the same. Because $12 \div 4 = 3$ and $12 \div 6 = 2$, then

$$\frac{1}{4} - \frac{1}{6} + \frac{5}{6} - \frac{11}{12} = \frac{1 \bullet 3}{4 \bullet 3} - \frac{1 \bullet 2}{6 \bullet 2} + \frac{5 \bullet 2}{6 \bullet 2} - \frac{11}{12}$$

$$= \frac{3}{12} - \frac{2}{12} + \frac{10}{12} - \frac{11}{12} = \frac{1}{12} + \frac{10}{12} - \frac{11}{12} = \frac{0}{12} = 0.$$

Ex. 5 $\frac{11}{38} - \frac{7}{57}$

Solution:

$38 = 2 \bullet 19$ and $57 = 3 \bullet 19$, so the L.C.D. = $2 \bullet 3 \bullet 19 = 114$.

Since $114 \div 38 = 3$ and $114 \div 57 = 2$, then

$$\frac{11}{38} - \frac{7}{57} = \frac{11 \bullet 3}{38 \bullet 3} - \frac{7 \bullet 2}{57 \bullet 2} = \frac{33}{114} - \frac{14}{114} = \frac{19}{114}$$

But, $\frac{19}{114}$ can be reduced: $\frac{19}{114} = \frac{1 \bullet 19}{6 \bullet 19} = \frac{1}{6}$

Objective b: Order of Operations

Simplify the following:

Ex. 6 $\frac{5}{14} + \frac{3}{11} \div \frac{7}{22}$

Solution:

Since division comes before addition, we must first invert & multiply:

$\frac{5}{14} + \frac{3}{11} \div \frac{7}{22} = \frac{5}{14} + \frac{3}{11} \bullet \frac{22}{7}$ (Reduce)

$= \frac{5}{14} + \frac{3}{1 \bullet 11} \bullet \frac{2 \bullet 11}{7} = \frac{5}{14} + \frac{3}{1} \bullet \frac{2}{7}$ (Multiply)

$= \frac{5}{14} + \frac{6}{7}$ (14 ÷ 7 = 2, so 14 is the L.C.D. Multiply $\frac{6}{7}$ by $\frac{2}{2}$)

$= \frac{5}{14} + \frac{6 \bullet 2}{7 \bullet 2} = \frac{5}{14} + \frac{12}{14} = \frac{17}{14}$ or $1\frac{3}{14}$

Ex. 7 $\frac{1}{6} \div \left(\frac{7}{10} - \frac{1}{5}\right)^2 + \frac{1}{4}$

Solution:

We need to start with the subtraction inside of the parenthesis.

$\frac{1}{6} \div \left(\frac{7}{10} - \frac{1}{5}\right)^2 + \frac{1}{4}$ (#1-parenthesis, #4-subtraction;

L.C.D of 10 and 5 is 10. Multiply $\frac{1}{5}$ by $\frac{2}{2}$)

$= \frac{1}{6} \div \left(\frac{7}{10} - \frac{1 \bullet 2}{5 \bullet 2}\right)^2 + \frac{1}{4}$

$= \frac{1}{6} \div \left(\frac{7}{10} - \frac{2}{10}\right)^2 + \frac{1}{4}$

$= \frac{1}{6} \div \left(\frac{5}{10}\right)^2 + \frac{1}{4}$ (Since $\frac{5}{10} = \frac{1 \bullet 5}{2 \bullet 5} = \frac{1}{2}$, then we get:)

$= \frac{1}{6} \div \left(\frac{1}{2}\right)^2 + \frac{1}{4}$ (#2-exponents; $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$)

$= \frac{1}{6} \div \frac{1}{4} + \frac{1}{4}$ (#3-division; invert and multiply)

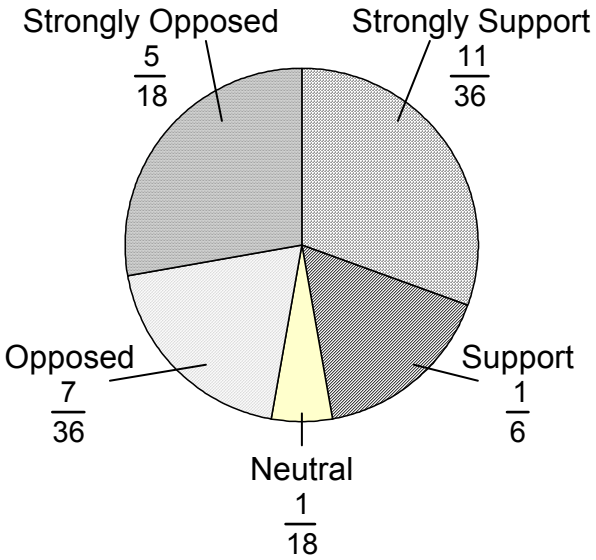
$= \frac{1}{6} \bullet \frac{4}{1} + \frac{1}{4} = \frac{1}{2 \bullet 3} \bullet \frac{2 \bullet 2}{1} + \frac{1}{4} = \frac{1}{3} \bullet \frac{2}{1} + \frac{1}{4}$

$= \frac{2}{3} + \frac{1}{4}$ (#4-addition; L.C.D is 12. Multiply $\frac{2}{3}$ by $\frac{4}{4}$ and $\frac{1}{4}$ by $\frac{3}{3}$)

$= \frac{2 \bullet 4}{3 \bullet 4} + \frac{1 \bullet 3}{4 \bullet 3} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

Objective c: Applications.

Ex. 8 A proposed greenhouse Gas emission tax would cost The average U.S. resident $300 per year and would reduce emissions by 1/16. A member of congress surveys her district to gauge its support for such a tax. The results are summarized on the right:



a) What fraction of the district surveyed is opposed or strongly opposed to the proposed tax?

b) If 4428 people were surveyed, how many were opposed or strongly opposed to the proposed tax?

Solution:

a) We need to add $\frac{5}{18}$ and $\frac{7}{36}$:

$\frac{5}{18} + \frac{7}{36}$ (L.C.D. = 36. Multiply $\frac{5}{18}$ by $\frac{2}{2}$. Then add)

$= \frac{5 \bullet 2}{18 \bullet 2} + \frac{7}{36} = \frac{10}{36} + \frac{7}{36} = \frac{17}{36}$

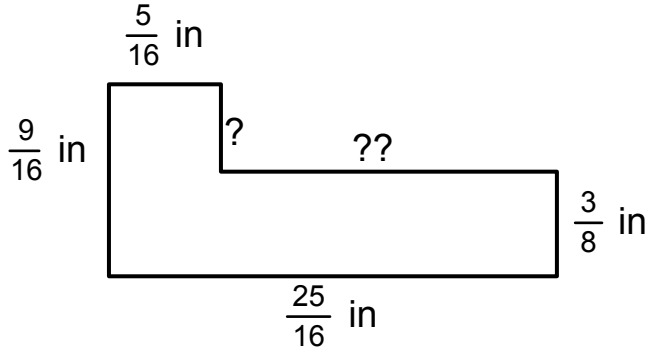
Thus, $\frac{17}{36}$ of the district is opposed or strongly opposed to the tax.

b) We need to multiply the answer from part a by 4428:

$\frac{17}{36} \bullet 4428 = \frac{17}{36} \bullet \frac{4428}{1} = \frac{17}{1 \bullet 36} \bullet \frac{123 \bullet 36}{1} = \frac{17}{1} \bullet \frac{123}{1} = 2091$

In the survey, 2091 people opposed or strongly opposed the tax.

Ex. 9 Find the missing sides and then calculate the perimeter.



Solution:

The first unknown side (?) $= \frac{9}{16} - \frac{3}{8} = \frac{9}{16} - \frac{3 \bullet 2}{8 \bullet 2} = \frac{9}{16} - \frac{6}{16} = \frac{3}{16}$ in

The second unknown side (??) $= \frac{25}{16} - \frac{5}{16} = \frac{20}{16} = \frac{4 \bullet 5}{4 \bullet 4} = \frac{5}{4}$ in

$P = \frac{9}{16} + \frac{5}{16} + \frac{3}{16} + \frac{5}{4} + \frac{3}{8} + \frac{25}{16} = \frac{9}{16} + \frac{5}{16} + \frac{3}{16} + \frac{5 \bullet 4}{4 \bullet 4} + \frac{3 \bullet 2}{8 \bullet 2} + \frac{25}{16}$

$= \frac{9}{16} + \frac{5}{16} + \frac{3}{16} + \frac{20}{16} + \frac{6}{16} + \frac{25}{16} = \frac{68}{16} = \frac{4 \bullet 17}{4 \bullet 4} = \frac{17}{4}$.

The perimeter is $\frac{17}{4}$ in or $4\frac{1}{4}$ in.