

Sect 3.4 - Adding and Subtracting Mixed Numbers

Objective a: Addition of Mixed Numbers.

When adding mixed numbers, we can work the problem two different ways. First, we can use the fact that addition is both commutative and associative and add the fraction parts of the mixed number and add the whole number parts. Or second, we could change the mixed numbers into improper fractions and work the problem like we did in section 3.3.

In the examples below, we will do the problem both ways so that we can compare. To check to see if our answer is reasonable, we will then round each number to the nearest whole number and perform the operation. Our answer should be close the exact answer.

Simplify the following. Then, estimate the answer:

Ex. 1 $4\frac{1}{5} + 6\frac{3}{8}$

Solution:

Mixed Number Method

The L.C.D. = 40

$$\begin{array}{r} 4\frac{1}{5} = 4\frac{1 \cdot 8}{5 \cdot 8} = 4\frac{8}{40} \\ + 6\frac{3}{8} = + 6\frac{3 \cdot 5}{8 \cdot 5} = + 6\frac{15}{40} \\ \hline 10\frac{23}{40} \end{array}$$

Estimate: $4\frac{1}{5} + 6\frac{3}{8}$

$\approx 4 + 6 = 10$ which

close to the exact answer.

Improper Fraction Method

First, rewrite the fractions as improper fractions:

$$\frac{4 \cdot 5 + 1}{5} + \frac{6 \cdot 8 + 3}{8} = \frac{21}{5} + \frac{51}{8}$$

The L.C.D. = 40

$$\begin{aligned} &= \frac{21 \cdot 8}{5 \cdot 8} + \frac{51 \cdot 5}{8 \cdot 5} \\ &= \frac{168}{40} + \frac{255}{40} = \frac{423}{40} \end{aligned}$$

$$\begin{array}{r} 10 \\ 40 \overline{) 423} \\ \underline{- 400} \\ 23 \end{array}$$

So, $\frac{423}{40} = 10\frac{23}{40}$.

Ex. 2 $7\frac{5}{8} + 3\frac{11}{12}$

Solution:

Mixed Number Method

The L.C.D. = 24

$$\begin{aligned} 7\frac{5}{8} &= 7\frac{5 \cdot 3}{8 \cdot 3} = 7\frac{15}{24} \\ + 3\frac{11}{12} &= + 3\frac{11 \cdot 2}{12 \cdot 2} = + 3\frac{22}{24} \\ \hline &10\frac{37}{24} \end{aligned}$$

But $\frac{37}{24}$ is an improper fraction,
so we will need to
convert it to a
mixed number.

$$\begin{array}{r} 1 \\ 24 \overline{) 37} \\ - 24 \\ \hline 13 \end{array}$$

Hence, $\frac{37}{24} = 1\frac{13}{24}$

$$\begin{aligned} \text{Thus, } 10\frac{37}{24} &= 10 + \frac{37}{24} \\ &= 10 + 1\frac{13}{24} = 11\frac{13}{24} \end{aligned}$$

Estimate: $7\frac{5}{8} + 3\frac{11}{12}$
 $\approx 8 + 4 = 12$ which is close.

Improper Fraction Method

First, rewrite the fractions as
improper fractions.

$$\frac{7 \cdot 8 + 5}{8} + \frac{3 \cdot 12 + 11}{12} = \frac{61}{8} + \frac{47}{12}$$

The L.C.D. = 24

$$= \frac{61 \cdot 3}{8 \cdot 3} + \frac{47 \cdot 2}{12 \cdot 2} = \frac{183}{24} + \frac{94}{24}$$

$$= \frac{183}{24} + \frac{94}{24} = \frac{277}{24}$$

$$\begin{array}{r} 11 \\ 24 \overline{) 277} \\ - 24 \\ \hline 37 \\ - 24 \\ \hline 13 \end{array}$$

So, $\frac{277}{24} = 11\frac{13}{24}$.

Ex. 3 $11 + 4\frac{3}{7}$

Solution:

Mixed Number Method

Simply add the whole number
to the whole number part of
the mixed number:

$$11 + 4\frac{3}{7} = 15\frac{3}{7}$$

Estimate: $11 + 4\frac{3}{7}$
 $\approx 11 + 4 = 15$ which is close.

Improper Fraction Method

Rewrite each number as an
improper fraction:

$$= \frac{11}{1} + \frac{31}{7} \quad \text{The L.C.D.} = 7$$

$$\text{So, } \frac{11 \cdot 7}{1 \cdot 7} + \frac{31}{7} = \frac{77}{7} + \frac{31}{7}$$

$$= \frac{108}{7} \quad \text{But } 108 \div 7 = 15 \text{ R } 3.$$

So, $\frac{108}{7} = 15\frac{3}{7}$.

Ex. 4 $6\frac{2}{3} + 5$

Solution:

Mixed Number Method

Simply add the whole number to the whole number part of the mixed number:

$$6\frac{2}{3} + 5 = 11\frac{2}{3}.$$

Estimate: $6\frac{2}{3} + 5$

$$\approx 7 + 5 = 12 \text{ which is close.}$$

Improper Fraction Method

Rewrite each number as an improper fraction:

$$= \frac{20}{3} + \frac{5}{1} \text{ The L.C.D.} = 3$$

$$\text{so, } \frac{20}{3} + \frac{5 \cdot 3}{1 \cdot 3} = \frac{20}{3} + \frac{15}{3}$$

$$= \frac{35}{3}. \text{ But } 35 \div 3 = 11 \text{ R } 2.$$

$$\text{So, } \frac{35}{3} = 11\frac{2}{3}.$$

Objective b: Subtraction of Mixed Numbers.

Ex. 5 $7\frac{5}{6} - 2\frac{3}{10}$

Solution:

Mixed Number Method

The L.C.D. = 30

$$7\frac{5}{6} = 7\frac{5 \cdot 5}{6 \cdot 5} = 7\frac{25}{30}$$

$$- 2\frac{3}{10} = - 2\frac{3 \cdot 3}{10 \cdot 3} = - 2\frac{9}{30}$$

$$5\frac{16}{30}$$

$$= 5\frac{2 \cdot 8}{2 \cdot 15}$$

$$= 5\frac{8}{15}$$

Estimate: $7\frac{5}{6} - 2\frac{3}{10}$

$$\approx 8 - 2 = 6$$

Improper Fraction Method

First, rewrite the fractions as improper fractions.

$$\frac{7 \cdot 6 + 5}{6} - \frac{2 \cdot 10 + 3}{10} = \frac{47}{6} - \frac{23}{10}$$

The L.C.D. = 30

$$= \frac{47 \cdot 5}{6 \cdot 5} - \frac{23 \cdot 3}{10 \cdot 3} = \frac{235}{30} - \frac{69}{30}$$

$$= \frac{166}{30} = \frac{2 \cdot 83}{2 \cdot 15} = \frac{83}{15}$$

$$\begin{array}{r} 5 \\ 15 \overline{) 83} \end{array}$$

$$\underline{- 75}$$

$$8$$

$$\text{So, } \frac{83}{15} = 5\frac{8}{15}.$$

Ex. 6 $9\frac{1}{3} - 4\frac{5}{6}$

Solution:

Mixed Number Method

The L.C.D. = 6

$$\begin{array}{r} 9\frac{1}{3} = 9\frac{1 \cdot 2}{3 \cdot 2} = 9\frac{2}{6} \\ - 4\frac{5}{6} = - 4\frac{5}{6} = - 4\frac{5}{6} \\ \hline \end{array}$$

Here, we cannot subtract $\frac{5}{6}$ from

$\frac{2}{6}$ so we will need to borrow one from the nine and change $1\frac{2}{6}$ into an improper fraction.

$$\begin{array}{r} 9\frac{2}{6} = 9\overset{8}{1}\frac{2}{6} = 8\frac{1 \cdot 6 + 2}{6} = 8\frac{8}{6} \\ - 4\frac{5}{6} = - 4\frac{5}{6} = - 4\frac{5}{6} = - 4\frac{5}{6} \\ \hline 4\frac{3}{6} \end{array}$$

$$= 4\frac{3 \cdot 1}{3 \cdot 2} = 4\frac{1}{2}.$$

Estimate: $9\frac{1}{3} - 4\frac{5}{6} \approx 9 - 5 = 4.$

Improper Fraction Method

First, rewrite the fractions as improper fractions.

$$\frac{9 \cdot 3 + 1}{3} - \frac{4 \cdot 6 + 5}{6} = \frac{28}{3} - \frac{29}{6}$$

The L.C.D. = 6

$$\begin{aligned} &= \frac{28 \cdot 2}{3 \cdot 2} - \frac{29}{6} = \frac{56}{6} - \frac{29}{6} \\ &= \frac{27}{6} \end{aligned}$$

$$\begin{array}{r} 4 \\ 6 \overline{) 27} \\ - 24 \\ \hline 3 \end{array}$$

$$\begin{aligned} \text{So, } \frac{27}{6} &= 4\frac{3}{6} = 4\frac{3 \cdot 1}{3 \cdot 2} \\ &= 4\frac{1}{2}. \end{aligned}$$

Ex. 7 $11\frac{7}{9} - 5$

Solution:

Mixed Number Method

Simply subtract the whole number from the whole number part of the mixed number:

$$11\frac{7}{9} - 5 = 6\frac{7}{9}.$$

Estimate: $11\frac{7}{9} - 5$

$$\approx 12 - 5 = 7.$$

Improper Fraction Method

Rewrite each number as an improper fraction:

$$= \frac{106}{9} - \frac{5}{1} \quad \text{The L.C.D.} = 9$$

$$\text{So, } \frac{106}{9} - \frac{5 \cdot 9}{1 \cdot 9} = \frac{106}{9} - \frac{45}{9}$$

$$= \frac{61}{9}. \quad \text{But } 61 \div 9 = 6 \text{ R } 7.$$

$$\text{So, } \frac{61}{9} = 6\frac{7}{9}.$$

Ex. 8 $15 - 6\frac{3}{8}$

Solution:

Mixed Number Method

We cannot simply subtract 6 from 15 since there is no fraction part to 15. We must borrow one 15 and rewrite the one as an improper fraction:

$$\begin{array}{r} 15 = 15^1 = 14\frac{1}{1} \\ - 6\frac{3}{8} = -6\frac{3}{8} = -6\frac{3}{8} \\ \hline \end{array}$$

The L.C.D. = 8, so

$$\begin{array}{r} 14\frac{1}{1} = 14\frac{1 \cdot 8}{1 \cdot 8} = 14\frac{8}{8} \\ - 6\frac{3}{8} = -6\frac{3}{8} = -6\frac{3}{8} \\ \hline 8\frac{5}{8} \end{array}$$

Estimate: $15 - 6\frac{3}{8} \approx 15 - 6 = 9.$

Improper Fraction Method

Rewrite each number as an improper fraction:

$$= \frac{15}{1} - \frac{51}{8} \quad \text{The L.C.D.} = 8$$

$$\text{So, } \frac{15 \cdot 8}{1 \cdot 8} - \frac{51}{8} = \frac{120}{8} - \frac{51}{8}$$

$$= \frac{69}{8}. \text{ But } 69 \div 8 = 8 \text{ R } 5.$$

$$\text{So, } \frac{69}{8} = 8\frac{5}{8}.$$

Objective c: Applications of Mixed Numbers.

Solve the following:

Ex. 9 On news of their last quarter earnings, the price per share of OverRun's stock fell $\$7\frac{3}{8}$ per share on Monday. If it started the day at $\$52\frac{1}{16}$ per share, find the price per share at the end of the day.

Solution:

Here, we need to subtract:

$$\begin{array}{r} 52\frac{1}{16} = 52\frac{1}{16} = 52\frac{1}{16} = 51\frac{17}{16} = 51\frac{17}{16} \\ - 7\frac{3}{8} = -7\frac{3 \cdot 2}{8 \cdot 2} = -7\frac{6}{16} = -7\frac{6}{16} = -7\frac{6}{16} \\ \hline \$44\frac{11}{16} \text{ per share} \end{array}$$

Ex. 10 A wedding cake recipe calls for $1\frac{3}{4}$ cups of sugar. If the recipe makes enough cake for 30 people, how many cups of sugar will be needed to make enough cake for 150 people?

Solution:

Take the total number of people and divided by the number of people one cake recipe can feed:

$$150 \div 30 = 5 \text{ cake recipes needed.}$$

Now take the amount of sugar needed for each cake recipe and multiply by the number of cake recipes needed:

$$5 \cdot 1\frac{3}{4} = \frac{5}{1} \cdot \frac{7}{4} = \frac{35}{4}.$$

But $\begin{array}{r} 8 \\ \hline 4 \overline{)35} \\ \underline{-32} \\ 3 \end{array}$

So, $8\frac{3}{4}$ cups of sugar are needed.

Ex. 11 Juanita has three presents that she needs to mail to her nieces and nephews for Christmas. She places the three presents in a box and packs them carefully so that the presents will survive the mail. If the first two presents weigh $9\frac{5}{16}$ pounds each and the third present weighs $11\frac{13}{16}$ pounds, how much will the package weigh altogether if the box and packing material weigh 2 pounds?

Solution:

Simply add all the weights:

$$\begin{array}{r} 9\frac{5}{16} \\ 9\frac{5}{16} \\ 11\frac{13}{16} \\ + 2 \\ \hline 31\frac{23}{16} \end{array}$$

$\begin{array}{r} 1 \\ \hline 16 \overline{)23} \\ \underline{-16} \\ 7 \end{array}$

$$31\frac{23}{16} = 31 + 1\frac{7}{16} = 32\frac{7}{16} \text{ pounds.}$$