Sect 3.5 - Order of Operations

Objective a: Applying the order of operations to fractions.

Recall the order of operations:

Order of Operations

1) Parentheses - Do operations inside of Parentheses ( ), [ ], { }, | |
2) Exponents including square roots.
3) Multiplication or Division as they appear from left to right.
4) Addition or Subtraction as they appear from left to right.

Simplify the following:

Ex. 1 $7\frac{1}{3} \div 3\frac{1}{7} \bullet 21 - 16\frac{2}{3}$

Solution:

Since there are no parentheses or exponents, we start with step #3, multiply or divide as they appear from left to right:

$7\frac{1}{3} \div 3\frac{1}{7} \bullet 21 - 16\frac{2}{3}$ (change $7\frac{1}{3}$ & $3\frac{1}{7}$ to improper fractions)

$= \frac{22}{3} \div \frac{22}{7} \bullet 21 - 16\frac{2}{3}$ (invert and change operation to •)

$= \frac{22}{3} \bullet \frac{7}{22} \bullet 21 - 16\frac{2}{3} = \frac{1 \bullet 22}{3} \bullet \frac{7}{22 \bullet 1} \bullet 21 - 16\frac{2}{3}$

$= \frac{1}{3} \bullet \frac{7}{1} \bullet 21 - 16\frac{2}{3}$ (#3-multiplication)

$= \frac{7}{3} \bullet 21 - 16\frac{2}{3}$ (write 21 over 1)

$= \frac{7}{3} \bullet \frac{21}{1} - 16\frac{2}{3} = \frac{7}{1 \bullet 3} \bullet \frac{3 \bullet 7}{1} - 16\frac{2}{3}$ (reduce)

$= \frac{7}{1} \bullet \frac{7}{1} - 16\frac{2}{3}$ (#3-multiplication)

$= 49 - 16\frac{2}{3}$ (borrow one from 49)

$= 48 \quad 1 - 16\frac{2}{3} = 48\frac{1}{1} - 16\frac{2}{3}$ (L.C.D. = 3)

$= 48\frac{1 \bullet 3}{1 \bullet 3} - 16\frac{2}{3} = 48\frac{3}{3} - 16\frac{2}{3}$ (#4-subtraction)

$= 32\frac{1}{3}$.

Ex. 2 $\left(1\frac{3}{4}\right)^2 \div 2\frac{4}{5} - \left(\frac{4}{\sqrt{9}} - 2\frac{3}{8} \bullet \frac{2^2}{19}\right)$

Solution:

Let's first change all the mixed numbers into improper fractions:

$\left(1\frac{3}{4}\right)^2 \div 2\frac{4}{5} - \left(\frac{4}{\sqrt{9}} - 2\frac{3}{8} \bullet \frac{2^2}{19}\right)$

$= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{\sqrt{9}} - \frac{19}{8} \bullet \frac{2^2}{19}\right)$ (#1-parentheses, #2-exponents)

$= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{3} - \frac{19}{8} \bullet \frac{4}{19}\right)$ (reduce inside the parentheses)

$= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{3} - \frac{1 \bullet 19}{2 \bullet 4} \bullet \frac{1 \bullet 4}{1 \bullet 19}\right)$

$= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{3} - \frac{1}{2} \bullet \frac{1}{1}\right)$ (#1-parentheses, #3-multiplication)

$= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{3} - \frac{1}{2}\right)$ (L.C.D. = 6)

$= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4 \bullet 2}{3 \bullet 2} - \frac{1 \bullet 3}{2 \bullet 3}\right)$

$= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{8}{6} - \frac{3}{6}\right)$ (#1-parentheses, #4-subtraction)

$= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \frac{5}{6}$ (#2-exponents)

$= \frac{49}{16} \div \frac{14}{5} - \frac{5}{6}$ (invert and change operation to •)

$= \frac{49}{16} \bullet \frac{5}{14} - \frac{5}{6}$ (reduce)

$= \frac{7 \bullet 7}{16} \bullet \frac{5}{7 \bullet 2} - \frac{5}{6}$

$= \frac{7}{16} \bullet \frac{5}{2} - \frac{5}{6}$ (#3-multiplication)

$= \frac{35}{32} - \frac{5}{6}$ (L.C.D. = 96)

$= \frac{35 \bullet 3}{32 \bullet 3} - \frac{5 \bullet 16}{6 \bullet 16} = \frac{105}{96} - \frac{80}{96}$ (#4-subtraction)

$= \frac{25}{96}$

Ex. 3 $\left(5\frac{1}{4}\bullet\frac{1}{6}-\frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24}-\frac{3}{11}$

Solution:

Change $5\frac{1}{4}$ into an improper fraction:

$\left(5\frac{1}{4}\bullet\frac{1}{6}-\frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24}-\frac{3}{11}$

$=\left(\frac{21}{4}\bullet\frac{1}{6}-\frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24}-\frac{3}{11}$ (reduce)

$=\left(\frac{3\bullet 7}{4}\bullet\frac{1}{3\bullet 2}-\frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24}-\frac{3}{11}$

$=\left(\frac{7}{4}\bullet\frac{1}{2}-\frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24}-\frac{3}{11}$ (#1-parentheses, #3-multiplication)

$=\left(\frac{7}{8}-\frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24}-\frac{3}{11}$ (#1-parentheses, #4-subtraction)

$=\left(\frac{6}{8}\right)^2 \div \frac{\sqrt{121}}{24}-\frac{3}{11}=\left(\frac{2\bullet 3}{2\bullet 4}\right)^2 \div \frac{11}{24}-\frac{3}{11}$ (reduce)

$=\left(\frac{3}{4}\right)^2 \div \frac{\sqrt{121}}{24}-\frac{3}{11}$ (#2-exponents)

$=\frac{9}{16} \div \frac{11}{24}-\frac{3}{11}$ (invert and change operation to $\bullet$)

$=\frac{9}{16} \bullet \frac{24}{11}-\frac{3}{11}=\frac{9}{2\bullet 8} \bullet \frac{3\bullet 8}{11}-\frac{3}{11}$ (reduce)

$=\frac{9}{2} \bullet \frac{3}{11}-\frac{3}{11}$ (#3-multiplication)

$=\frac{27}{22}-\frac{3}{11}$ (L.C.D. = 22)

$=\frac{27}{22}-\frac{3\bullet 2}{11\bullet 2}=\frac{27}{22}-\frac{6}{22}$ (#4-subtraction)

$=\frac{21}{22}$.

Objective b: Applications.

Solve the following:

Ex. 4 If twenty-four pounds of candy is distributed in $\frac{3}{8}$-pound bags, how many bags will be filled?

Solution:

Distributed means divide up so we need to take the total candy divide by the amount of candy in each bag;

$24 \div \frac{3}{8} = \frac{24}{1} \bullet \frac{8}{3} = \frac{3 \bullet 8}{1} \bullet \frac{8}{3 \bullet 1} = \frac{8}{1} \bullet \frac{8}{1} = 64$

Sixty-four bags can be filled.

Ex. 5 A developer has 250 acres of land in the hill country that she is planning to develop into a new subdivision called Saddle Rock. If $30\frac{1}{12}$ acres of land will be reserved for roads and utilities, how many houses can she build in the subdivision if she wants the average lot size to be $2\frac{5}{12}$ acres?

Solution:

First, subtract $30\frac{1}{12}$ from 250 to find the number of acres that will be used for houses:

$$\begin{array}{ccccccccc} 250 & = & 249\ 1 & = & 249\ \frac{1}{1} & = & 249\frac{1 \bullet 12}{1 \bullet 12} & = & 249\frac{12}{12} \\ -30\frac{1}{12} & = & -30\frac{1}{12} & = & -30\frac{1}{12} & = & -30\frac{1}{12} & = & -30\frac{1}{12} \\ \hline & & & & & & & & 219\frac{11}{12} \text{ Acres} \end{array}$$

Now, take the total acres used for houses and divide it by $2\frac{5}{12}$:

$219\frac{11}{12} \div 2\frac{5}{12} = \frac{2639}{12} \div \frac{29}{12} = \frac{2639}{12} \bullet \frac{12}{29} = \frac{29 \bullet 91}{1 \bullet 12} \bullet \frac{1 \bullet 12}{1 \bullet 29}$

$= \frac{91}{1} \bullet \frac{1}{1} = 91$ houses.

Ex. 6 A countertop is made of $\frac{11}{16}$-in particleboard and is covered with $\frac{1}{4}$-in laminated plastic. If the length of the edge is 132 in, how many square inches of the metal edging are needed?

Solution:

To find the width of the metal edging, we need to add the thickness of the particleboard and the plastic:

$$\frac{11}{16} + \frac{1}{4} = \frac{11}{16} + \frac{1 \bullet 4}{4 \bullet 4} = \frac{11}{16} + \frac{4}{16} = \frac{15}{16} \text{ in}$$

To find the area, we multiply the length times the width:

$$\frac{15}{16} \bullet 132 = \frac{15}{16} \bullet \frac{132}{1} = \frac{15}{4 \bullet 4} \bullet \frac{4 \bullet 33}{1} = \frac{15}{4} \bullet \frac{33}{1} = \frac{495}{4} \text{ or } 123\frac{3}{4}$$

The metal edging will be $123\frac{3}{4}$ in^2.