Sect 4.1 - Introduction to Decimals

Objective a: Understanding Place Values and Decimals

In chapter 1, we discussed the place value of the digits in a number. In a number like 3,278, we can say that 3 is in the thousands place, 2 in the hundreds place, 7 is in the tens place, and 8 is in the ones place. The 3 is worth 3•1000, the 2 is worth 2•100, the 7 is worth 7•10, and the 8 is worth 8•1. As we move from left to right, the value of the digit decreases by a factor of 10 (i.e., we are dividing by 10 each time we move a place to the right). In this chapter, we want to represent quantities smaller than one using our place value system. We will use a period called a decimal point to indicate where the whole number part of a number ends and the portion of the number that is smaller than one begins. The first digit to the right of the ones place (the first after the decimal point) will have a place value of 1 ÷ 10 or $\frac{1}{10}$. We will call this the ten***th***s place. The next digit to the right will have a place value of $\frac{1}{10} \div 10 = \frac{1}{10} \bullet \frac{1}{10} = \frac{1}{100}$. We will call this the hundred***th***s place. And we will continue this pattern. So, in number like 3,278.5694, the digits would have the following place values:

3,	**2**	**7**	**8**	**.**	**5**	**6**	**9**	**4**
THOUSANDS	HUNDREDS	TENS	ONES	DECIMAL POINT	TENTHS	HUNDREDTHS	THOUSANDTHS	TENS THOUSANDTHS

Identify the place value of each underlined digit:

Ex. 1a 56.78$\underline{9}$53

Ex. 1b 0.9823$\underline{4}$2

Ex. 1c 456.7$\underline{8}$34

Ex. 1d 32$\underline{7}$8.5694

Solution:

a) Since 9 is the third digit to the right, its place value is 1 over 1 followed by three zeros or the thousandths place.

b) Since 4 is the fifth digit to the right, its place value is 1 over 1 followed by five zeros or the hundred-thousandths place.

c) Since 8 is the second digit to the right, its place value is 1 over 1 followed by two zeros or the hundredths place.

d) Since 7 is the second digit to the **left**, its place value is 1 followed by two zeros or the tens place.

If we were to write this in expanded form, it would look like this:

$3{,}278.5694 = 3000 + 200 + 70 + 8 + \frac{5}{10} + \frac{6}{100} + \frac{9}{1000} + \frac{4}{10000}$.

Notice that the number of places that the digit is to the right of the decimal point corresponds to the number of zeros that follows the one in the denominator in the expanded form. For example, the 1 digit is in the third place to the right of the decimal point so it is worth 9 ÷ 1**000**. This gives us a natural way of writing decimals:

Write the following as a fraction and as a decimal:

Ex. 2 Seven tenths

Solution:

Think of 7 over 10: $\frac{7}{10}$

Ten has one zero, so the seven is in the first place to the right of the decimal point: 0.7

Ex. 3 Fifty-three hundredths

Solution:

Think of 53 over 100: $\frac{53}{100}$

One hundred has two zeros, so the fifty-three has to occupy the first two places to the right of the decimal point: 0.53

Ex. 4 Nine thousandths

Solution:

Think of 9 over 1000: $\frac{9}{1000}$

One thousand has three zeros, so the nine is in the third place to the right of the decimal point: 0.009

In mathematics, we use the word "and" to indicate the decimal point.

Write the following in decimal notation:

Ex. 5 Fourteen and twenty-three hundredths

Solution:

14 goes to the left of the decimal point, 23 to the right: 14.23

Ex. 6 One hundred five and seventy-three ten-thousandths

Solution:

105 goes to the left of the decimal point and 73 to the right, but since 10000 has four zeros, the 73 will occupy the third and fourth digits to the right of the decimal point: 105.0073

Ex. 7 Twenty-eight and one hundred seven thousandths

Solution:

28 goes to the left of the decimal point and 107 goes to the right: 28.107

Write the following numbers in words:

Ex. 8 8.2

Solution:

8 is to the left of the decimal and 2 is in the tenth place to the right of the decimal:

Eight and two tenths.

Ex. 9 19.042

Solution:

19 is to the left of the decimal and 42 is to the right of the decimal. Since there are three digits to the right of the decimal point, then last digit 2 is in the thousandths place:

Nineteen and forty-two thousandths.

Objective b: Writing Decimals as fractions or mixed numbers.

When converting decimals into fractions, it is easy to reduce since the denominator will always be a power of ten. Look for factors of 2 and 5 when reducing since the power of ten consist of only powers of twos and fives. If neither 2 nor 5 will divide the numerator evenly, then the fraction is already reduced to lowest terms.

Write the following decimals as fractions or as a mixed number:

Ex. 10 0.9

Solution:

Since there is one digit to the right of the decimal point, we write nine over 1 followed by one zero:

$$\frac{9}{10}$$

Ex. 11 0.068

Solution:

Since there are three digits to the right of the decimal point, we write 68 over 1 followed by three zeros and reduce:

$$\frac{68}{1000} = \frac{17}{250}$$

Ex. 12 9.23

Solution:

The nine is the whole number part. Since there are two digits to the right of the decimal point, we write the result over 1 followed by two zeros:

$$\frac{923}{100} \text{ or } 9\frac{23}{100}$$

Ex. 13 8.085

Solution:

The eight is the whole number part. Since there are three digits to the right of the decimal point, we write the result over 1 followed by three zeros and reduce:

$$\frac{8085}{1000} = \frac{1617}{200}$$

$$8\frac{85}{1000} = 8\frac{17}{200}$$

Objective c: Comparing two decimals.

In a decimal like 2.34, two is in the ones place, three is in the tenths place and four is in the hundredths place. For the place values beyond the hundredths place, there are zeros. So, 2.34 is the same as 2.340000... We usually do not write the extra zeros beyond the last non-zero digit to the right of the decimal point. The exceptions would be when we write money and in science, when we want to indicate the accuracy in measurements. In money, we might write the cost of an item as $2.50 and in science, if we want to indicate the measurement of length to be accurate to the nearest thousandth, we might write the measurement as 7.400 meters. In comparing decimals, it is sometimes useful to write the extra zeros beyond the last non-zero digit so that the numbers have the same number of digits to right of the decimal point. Then, if we pretend that there is no decimal point, we can easily see which number is larger.

Compare using <, >, or =:

Ex. 14 2.6 2.58

Solution:
Write 2.6 as 2.60: 2.60 2.58
Since 260 > 258, then 2.6 > 2.58.

Ex. 15 0.003199 0.0032

Solution:
Write 0.0032 as 0.003200: 0.003199 0.003200
Since 3199 < 3200, then 0.003199 < 0.0032.

Ex. 16 2.006 2.0056

Solution:
Write 2.006 as 2.0060: 2.0060 2.0056
Since 20060 > 20056, then 2.006 > 2.0056.

Objective d: Rounding Decimals.

Recall the rules for rounding from chapter 1:

Rules for Rounding:

1) If the first digit to the right of the round-off digit is less than 5, keep the round-off digit the same. If the first digit to the right of the round-off digit is 5 or greater, add one to the round off digit.

2) Replace every digit to the right of the round-off digit by zeros.

To this procedure, we will add that at the end of the problem, we will drop all the zeros to the right of the last non-zero digit to the right of the decimal point.

Round each number to the indicated place value:

Ex. 17 7.638 to the nearest tenth.

Solution:
The round-off digit 6 is in the tenths place. 7.6̲38 ↑
The digit immediately to the right of the round-off digit is 3. Since this is smaller than 5, we will keep the six the same and replace all the digits to the right of 6 by zeros and then drop the extra zeros:

$7.638 \approx 7.600 = 7.6$.

Ex. 18 12.9382 to the nearest hundredth.

Solution:
The round-off digit 3 is in the hundredths place. 12.93̲82 ↑
The digit immediately to the right of the round-off digit is 8. Since this is 5 or larger, we will add one to the three and replace all the digits to the right of 3 by zeros and then drop the extra zeros:

$12.9382 \approx 12.9400 = 12.94$

Round off the following numbers to the indicated place value:

Ex. 19

Number	Thousandths	Hundredths	Tenths
8.10263			
4.93796			
0.9999			
0.0352			
208.7462			
0.0076			

Solution:

Number	Thousandths	Hundredths	Tenths
8.10263	≈ 8.103	≈ 8.10 = 8.1	≈ 8.1
4.93796	≈ 4.938	≈ 4.94	≈ 4.9
0.9999	≈ 1.000 = 1	≈ 1.00 = 1	≈ 1.0 = 1
0.0352	≈ 0.035	≈ 0.04	≈ 0.0 = 0
208.7462	≈ 208.746	≈ 208.75	≈ 208.7
0.0076	≈ 0.008	≈ 0.01	≈ 0.0 = 0