Sect 4.4 - Dividing Decimals

Objective a: Division of Decimals.

Recall that in dividing whole numbers, we were careful about lining the place values of the quotient with the dividend. If we are dividing a decimal by a whole number, we do exactly the same thing.

Simplify the following and then estimate the answer:

Ex. 1 $76.24 \div 8$

Solution:

```
   9
8 | 76.24
 – 72
    4 2
```

8 does not go into 7, but it does go into 76 nine times. We write 9 above the 6 and multiply 8 and 9 to get 72. Subtract 72 from 76 and bring down the 2. Now, repeat the process.

```
   9.5
8 | 76.24
 – 72
    4 2
  – 40
     24
```

8 goes into forty-two five times. We write 5 above the 2 and multiply 8 and 5 to get 40. Subtract 40 from 42 and bring down the 4. Now, repeat the process.

```
   9.53
8 | 76.24
 – 72
    4 2
  – 40
     24
   – 24
      0
```

8 goes into twenty-four three times. We write 3 above the 4 and multiply 8 and 3 to get 24. Subtract 24 from 24 to get zero. The remainder is zero. So the answer is 9.53.

Estimate: 80 ÷ 8 = 10.

Recall that a number like 76.24 is the same as 76.240000... If the division does not work out evenly, we can bring down a zero and continue the process. If it still does not divide evenly, we can continue to bring down zeros after each step to do the division. We will stop when we get a remainder of zero or when the numbers repeat.

Simplify the following and then estimate the answer:

Ex. 2 $5.123 \div 5$

Solution:

$$
\begin{array}{r|l}
5 & 5.123\textit{0} \\
 & -\ 5 \\
 & \quad 0\ 12 \\
 & -\ 10 \\
 & \quad\ \ 23 \\
 & -\ 20 \\
 & \quad\ \ 3\textit{0} \\
 & -\ 30 \\
 & \quad\ \ 0
\end{array}
$$

Take 5 divided by 5 to get 1. Write down 1.
1•5 = 5, 5 – 5 = 0. Bring down the 1.
Five goes into 1 zero times. Write down 0.
Bring down the 2. Five goes into 12 twice.
Write down the 2. 2•5 = 10, 12 – 10 = 2.
Bring down the 3. Five goes into 23 four times
Write down the 4. 4•5 =20, 23 – 20 = 3.
Bring down a *0*. Five goes into 30 six times.
6•5 = 30, 30 – 30 = 0.

The answer is 1.0246. *Estimate: 5 ÷ 5 = 1.*

Ex. 3 $0.7412 \div 3$

Solution:

$$
\begin{array}{r|l}
3 & 0.7412 \\
 & -\ 6 \\
 & \quad\ \ 14 \\
 & -\ 12 \\
 & \quad\ \ 21 \\
 & -\ 21 \\
 & \quad\ \ 02\textit{0} \\
 & -\ 18 \\
 & \quad\ \ \textit{20}
\end{array}
$$

Three goes into 7 twice. Write down the 2.
2•3 = 6, 7 – 6 = 1. Bring down the 4.
Three goes into 14 four times. Write down the 4.
4•3 = 12, 14 – 12 = 2. Bring down the 1
Three goes into 21 seven times. Write down the 7.
7•3 = 21, 21 – 21 = 0. Bring down the 2.
Three goes into 2 zero times. Bring down a zero.
Three goes into 20 six times. 6•3 = 18, 20 – 18 = 2.
Bring down a zero. Notice we get 20 again so this pattern will repeat.

In mathematics, the way we indicate that a digit or digits repeat is that we write the digit or digits once and put a line over the digit or digits. So, we will write the answer as $0.24\overline{706}$.

Estimate: 0.7 ÷ 3

$$
\begin{array}{r|l}
3 & 0.7\textit{00} \\
 & -\ 6 \\
 & \quad\ \ \textit{10} \\
 & -\ 9 \\
 & \quad\ \ \textit{10}
\end{array}
$$

$0.7 \div 3 = 0.2\overline{3}$.

We can divide by a decimal by first converting the decimal into a whole number. We can do this by multiplying by the appropriate power of 10. Whatever we do to the divisor, we also need to do the dividend.

Ex. 4 $6.58 \div 0.7$

Solution:

First, rewrite $6.58 \div 0.7 = \frac{6.58}{0.7}$. Since 0.7 has one place to the right of the decimal, we will need to multiply 0.7 by 10. Whatever we do to the bottom, we need to do the same thing to the top:

$\frac{6.58 \bullet 10}{0.7 \bullet 10} = \frac{65.8}{7}$. Now, divide $65.8 \div 7$:

$$\begin{array}{r} 9.4 \\ 7\overline{)\,65.8} \\ -\,63 \\ \hline 2\,8 \\ -\,2\,8 \\ \hline 0 \end{array}$$

A shortcut is to realize that the number of places that the decimal point gets moved in the divisor is the same number of place that the decimal point gets moved in the dividend. So, we can count the number of places the decimal point gets moved in the divisor and move the decimal point the same number of places in the dividend. *Estimate: 7 ÷ 0.7 = 10.*

Ex. 5 $0.0728 \div 0.056$

Solution:

We need to move the decimal point three places to the right to make 0.056 as a whole number. So, we need to do the same thing to the dividend:

$$0.056\overline{)\,0.0728} \quad \Rightarrow \quad 56\overline{)\,72.8}$$

$$\begin{array}{r} 1.3 \\ 56\overline{)\,72.8} \\ -\,56 \\ \hline 16\,8 \\ -\,16\,8 \\ \hline 0 \end{array}$$

Estimate: 0.07 ÷ 0.06

$$0.06\overline{)\,0.07} \quad \Rightarrow \quad 6\overline{)\,7.0}$$

$$\begin{array}{r} 1.166.... \\ 6\overline{)\,7.0} \\ -\,6 \\ \hline 1\,0 \\ -\,6 \\ \hline 4\,0 \\ -\,36 \\ \hline 4\,0 \end{array}$$

$0.07 \div 0.06 = 1.1\overline{6}$.

Ex. 6 $8113 \div 0.07$

Solution:

We need to move the decimal point two places to the right. Since 8113 is a whole number, the decimal point is after the 3.

$$0.07\overline{)8113.} \quad \Rightarrow \quad 7\overline{)811300}\ 115900$$

-7

11

-7

41

-35

63

-63

0

So, the answer is 115,900

Estimate: 8000 ÷ 0.07

= 114285.714285

Simplify:

Ex. 8 $0.68 \div 2.2$

Solution:

We need to move the decimal point one place to the right.

$$2.2\overline{)0.68} \quad \Rightarrow \quad 22\overline{)6.8}\ 0.30909...$$

-66

20

-0

200

-198

2

Etc.

Since two digits repeat, we will write the line over the two digits. So, the answer is $0.3\overline{09}$.

Ex. 9 $\frac{1}{4}$

Solution:

$\frac{1}{4} = 1 \div 4$, so we can do this as a division problem:

$$4\overline{)1.0}\ 0.25$$

-8

20

-20

0

So, the answer is 0.25.

The last example shows us how to convert a fraction into a decimal. We just divide the denominator into the numerator.

Ex. 10 $\frac{7}{2}$

Solution:

$\frac{7}{2} = 7 \div 2$, so we can divide the numerator by the denominator:

$$\begin{array}{r} 3.5 \\ 2\overline{)\,7.0} \\ -\,6 \\ \hline 10 \\ -\,10 \\ \hline 0 \end{array}$$

So, the answer is 3.5.

Objective b: Rounding a Quotient.

Divide and round your answer to the nearest hundredth:

Ex. 11 $\frac{3.45}{1.7}$

Solution:

Since the directions say to round to the nearest hundredth, we will need to carry out the division one digit further to the right of the round off digit. First, move the decimal point over one spot and then divide:

$$\begin{array}{r} 2.029 \\ 17\overline{)\,34.5} \\ -\,34 \\ \hline 50 \\ -\,34 \\ \hline 160 \\ -\,153 \\ \hline 7 \end{array}$$

Rounding to the nearest hundredth, we get ≈ 2.03.

Ex. 12 The quotient of 9.278 and 0.29

Solution:

Since the directions say to round to the nearest hundredth, we will need to carry out the division one digit further to the right of the round off digit. First, move the decimal point over two places and then divide:

```
   31.993
29 | 927.8
   – 87
     57
   – 29
     288
   – 261
      270
   – 261
        90
    – 87
         3
```

Rounding to the nearest hundredth, we get ≈ 31.99.

Ex. 13 $4 + 0 \div 5.8$

Solution:

Since 0 divided by any non-zero number is zero, then

$4 + 0 \div 5.8 = 4 + 0 = 4$.

Ex. 14 $3 + 9.62 \div 0$

Solution:

Since division by zero in undefined, then

$3 + 9.62 \div 0$ is undefined.

Objective c: Applications.

Solve the following:

Ex. 15 If Juan used 14.7 gallons of gas to drive 516 miles to El Paso, how many miles (to the nearest mile) per gallon did his car get?

Solution:

We need divide the number miles (516) by the number of gallons (14.7). Moving the decimal point one place to the right, we get:

```
   35.1 ≈ 35
147 | 5160.0
   – 441
     750
   – 735
     150
   – 147
       3
```

Juan's car got about 35 miles per gallon on the trip.

Ex. 16 The voltage output of a particular solar cell is 3.375 volts. If it is connected to circuit with 750–ohm load, find the current in amperes by finding the quotient of the voltage and the load.

Solution:

Divide 3.375 volts by 750 ohms:

$$
\begin{array}{r|l}
750 & 3.3750 \\
 & \underline{-\ 3\ 000} \\
 & 3750 \\
 & \underline{-\ 3750} \\
 & 0
\end{array}
\qquad 0.0045
$$

The current is 0.0045 amperes.

Ex. 17 For her five restaurants, Amanda purchases cheese for making cheeseburgers at \$2.15 per pound. She determines that 570 pounds of cheese is enough to make 15,000 cheeseburgers. What is the cost (to the nearest tenth of a cent) of the cheese per cheeseburger?

Solution:

First, we need to find the total cost of the cheese and then divide it by the number of cheeseburgers:

$(\$2.15 \bullet 570) \div 15{,}000$

$$
\begin{array}{rcrcr}
\textit{1}\ \textit{3} & & & \textit{2} & \\
2.15 & & 2.15 & & 2.15 \\
\times\ 570 & \Rightarrow & \times\ 570 & \Rightarrow & \times\ 570 \\
\hline
15050 & & 15050 & & 15050 \\
 & & 107500 & & +\ 107500 \\
\hline
 & & & & 1225.50
\end{array}
$$

$(\$2.15 \bullet 570) \div 15{,}000 = 1225.50 \div 15{,}000$

$$
\begin{array}{r|l}
15000 & 1225.5000 \\
 & \underline{-\ 120000} \\
 & 25500 \\
 & \underline{-\ 15000} \\
 & 105000 \\
 & \underline{-\ 105000} \\
 & 0
\end{array}
\qquad 0.0817 \approx 0.082
$$

The cheese will cost ≈ \$0.082 or 8.2¢ per cheeseburger.