Sect 4.5 - Converting Fractions into Decimals

Objectives a & b: Converting between fractions to decimals

Previously, we saw that a fraction like $\frac{1}{2}$ can be written as $1 \div 2$. Thus. we can perform the division like we did before:

$$\begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ -1\,0 \\ \hline 0 \end{array}$$

In general, to convert a fraction to a decimal, divide the numerator by the denominator:

$$\text{Denominator}\overline{)\text{Numerator}}$$

Convert the following fractions into decimals:

Ex. 1 $\frac{7}{8}$

Solution:

$$\begin{array}{r} 0.875 \\ 8\overline{)7.0} \\ -6\,4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

So, the answer is 0.875.

Ex. 2 $\frac{7}{11}$

Solution:

$$\begin{array}{r} 0.6363.... \\ 11\overline{)7.0} \\ -6\,6 \\ \hline 40 \\ -33 \\ \hline 70 \\ \text{Repeats} \end{array}$$

So, the answer is $0.\overline{63}$.

Ex. 3 $\frac{21}{5}$

Solution:

$$\begin{array}{r} 4.2 \\ 5\overline{)21.0} \\ -20 \\ \hline 1\,0 \\ -1\,0 \\ \hline 0 \end{array}$$

The answer is 4.2.

Ex. 4 $\frac{5}{9}$

Solution:

$$\begin{array}{r} 0.55.... \\ 9\overline{)5.0} \\ -4\,5 \\ \hline 5 \\ \text{Repeats} \end{array}$$

The answer is $0.\overline{5}$.

To convert a mixed number to a decimal, we will change the proper fraction of the mixed number into a decimal and then add the whole number part to the result. Let's try some examples:

<u>Convert the following into a decimal:</u>

Ex. 5 $4\frac{2}{3}$

<u>Solution:</u>

Convert the proper fraction to a decimal:

$$\begin{array}{r} 0.6666\ldots \\ 3\,\overline{)\,2.0} \\ -\,1\,8 \\ \hline 20 \end{array}$$

Repeats

Now add 4.

So, the answer is $4.\overline{6}$.

Ex. 6 $8\frac{5}{16}$

<u>Solution:</u>

Convert the proper fraction to a decimal:

$$\begin{array}{r} 0.3125 \\ 16\,\overline{)\,5.0} \\ -\,4\,8 \\ \hline 20 \\ -\,16 \\ \hline 40 \\ -\,32 \\ \hline 80 \\ -\,80 \\ \hline 0 \end{array}$$

Now add 8.

So, the answer is 8.3125.

Ex. 7 $2\frac{4}{7}$

<u>Solution:</u>

Convert the proper fraction to a decimal:

$$\begin{array}{r} 0.571428571428\ldots \\ 7\,\overline{)\,4.0} \\ -\,3\,5 \\ \hline 50 \\ -\,49 \\ \hline 10 \\ -\ \ 7 \\ \hline 30 \\ -\,28 \\ \hline 20 \\ -\,14 \\ \hline 60 \\ -\,56 \\ \hline 4 \end{array}$$

Repeats

Now, add 2 to our result. Since 571428 repeats, our answer is $2.\overline{571428}$.

Here are some common repeating decimals and their fractional equivalence:

$\frac{1}{9} = 0.\overline{1}$ $\frac{2}{9} = 0.\overline{2}$ $\frac{1}{3} = 0.\overline{3}$ $\frac{4}{9} = 0.\overline{4}$ $\frac{5}{9} = 0.\overline{5}$ $\frac{2}{3} = 0.\overline{6}$

$\frac{7}{9} = 0.\overline{7}$ $\frac{8}{9} = 0.\overline{8}$

Objective c: Comparing Decimals

Compare using $<$, $>$, or $=$:

Ex. 8 0.778 $\frac{7}{9}$

Solution:

Convert the fraction into a decimal:

$$\begin{array}{r}
0.777\ldots \\
9 \overline{)\,7.0} \\
\underline{-\,63} \\
70 \\
\underline{-\,63} \\
70 \\
\underline{-\,63} \\
7
\end{array}$$

Since 0.778 > 0.777...,

then 0.778 > $\frac{7}{9}$.

Ex. 9 2.142 $\frac{15}{7}$

Solution:

Convert the fraction into a decimal:

$$\begin{array}{r}
2.1428\ldots \\
7 \overline{)\,15.0} \\
\underline{-\,14} \\
10 \\
\underline{-\,7} \\
30 \\
\underline{-\,28} \\
20 \\
\underline{-\,14} \\
60 \\
\underline{-\,56} \\
4
\end{array}$$

Since 2.142 < 2.1428...,

then 2.142 < $\frac{15}{7}$.

Ex. 10 0.372 $\frac{22}{59}$

Solution:

Convert $\frac{22}{59}$ into a decimal

by dividing 22 by 59:

22 ÷ 59 = 0.3728...

But, this is larger than 0.372 since 0.372 = 0.3720.

So, 0.372 < $\frac{22}{59}$.

Ex. 11 12.715 $\frac{89}{7}$

Solution:

Convert $\frac{89}{7}$ into a decimal

by dividing 89 by 7:

89 ÷ 7 = 12.714...

But this is smaller than 12.715.

So, 12.715 > $\frac{89}{7}$.

List in Order from Smallest to Largest:

Ex. 12 $\quad 0.629, \frac{5}{8}, 0.65, \frac{7}{10}$

Solution:

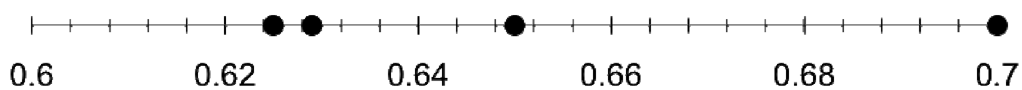
Since $\frac{5}{8} = 0.625$ and $\frac{7}{10} = 0.7$
and $0.625 < 0.629 < 0.65 < 0.7$,
then $\frac{5}{8}$, 0.629, 0.65, $\frac{7}{10}$.

Ex. 13 $\quad 0.047, 0.151, \frac{1}{200}, \frac{3}{20}$

Solution:

Since $\frac{1}{200} = 0.005$ & $\frac{3}{20} = 0.15$
and $0.005 < 0.047 < 0.15 < 0.151$,
then $\frac{1}{200}$, 0.047, $\frac{3}{20}$, 0.151.

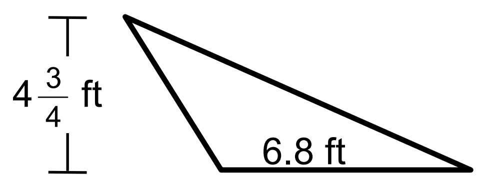
Notice the numbers in example 12 are very close together. If we needed to plot these numbers on a number line, we would need to blow up the scale in order to see the different locations of the numbers:



Objective d: $\quad$ Applications

Find the area of the following (Exact Value Only):

Ex. 14



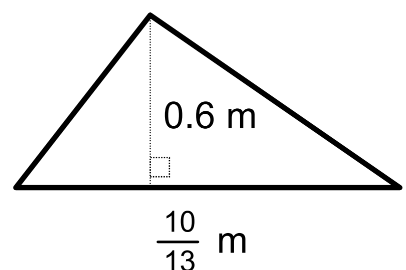
Solution:

Since $\frac{1}{2} = 0.5$ and $\frac{3}{4} = 0.75$
are nice, terminating decimals,
we will do this as a decimal
problem:

$A = \frac{1}{2} \bullet (6.8) \bullet (4\frac{3}{4})$

$= 0.5 \bullet 6.8 \bullet 4.75 = 16.15 \text{ ft}^2$.

Ex. 15



Solution:

Since $\frac{10}{13} = 0.768230...$ is
not a terminating decimal,
we will do this as a fraction
problem: 0.6 = 6/10 = 3/5,

so, $A = \frac{1}{2} \bullet \frac{10}{13} \bullet 0.6 = \frac{1}{2} \bullet \frac{10}{13} \bullet \frac{3}{5}$

$= \frac{1}{2} \bullet \frac{2 \bullet 5}{13} \bullet \frac{3}{5} = \frac{1}{1} \bullet \frac{1}{13} \bullet \frac{3}{1}$

$= \frac{3}{13} \text{ m}^2$.