

Sect 4.6 - Order of Operations

Objective a: Applying the order of operations to decimals.

Recall the order of operations:

Order of Operations

- 1) Parentheses - Do operations inside of Parentheses (), [], { }, | |
- 2) Exponents including square roots.
- 3) Multiplication or Division as they appear from left to right.
- 4) Addition or Subtraction as they appear from left to right.

Simplify the following:

Ex. 1 $4.8 \bullet 1.6 \div 0.8 \bullet 0.4$

Solution:

Since there are no parentheses or exponents, we start with step #3, multiply or divide as they appear from left to right:

$$\begin{aligned}
 &4.8 \bullet 1.6 \div 0.8 \bullet 0.4 && \text{(#3-multiplication)} \\
 &= 7.68 \div 0.8 \bullet 0.4 && \text{(#3-division)} \\
 &= 9.6 \bullet 0.4 && \text{(#3-multiplication)} \\
 &= 3.84.
 \end{aligned}$$

Ex. 2 $5.1 - 3.4 \div 1.7 + (\sqrt{25} - 3.5)^2$

Solution:

$$\begin{aligned}
 &5.1 - 3.4 \div 1.7 + (\sqrt{25} - 3.5)^2 && \text{(#1-parentheses, #2-expon.)} \\
 &5.1 - 3.4 \div 1.7 + (5 - 3.5)^2 && \text{(#1-parentheses, #4-subtraction)} \\
 &= 5.1 - 3.4 \div 1.7 + (1.5)^2 && \text{(#2-exponents)} \\
 &= 5.1 - 3.4 \div 1.7 + 2.25 && \text{(#3-division)} \\
 &= 5.1 - 2 + 2.25 && \text{(#4-subtraction)} \\
 &= 3.1 + 2.25 && \text{(#4-addition)} \\
 &= 5.35
 \end{aligned}$$

Ex. 3 $0.3[7(1.2)^2 - \sqrt{0.49}(10)] \div 0.2 - 0.72 \div 0.3$

Solution:

$$\begin{aligned}
 &0.3[7(1.2)^2 - \sqrt{0.49}(10)] \div 0.2 - 0.72 \div 0.3 \quad (\#1\text{-parent.}, \#2\text{-exp.}) \\
 &= 0.3[7(1.44) - (0.7)(10)] \div 0.2 - 0.72 \div 0.3 \quad (\#1\text{-parent.}, \#3\text{-mult.}) \\
 &= 0.3[10.08 - (0.7)(10)] \div 0.2 - 0.72 \div 0.3 \quad (\#1\text{-parent.}, \#3\text{-mult.}) \\
 &= 0.3[10.08 - 7] \div 0.2 - 0.72 \div 0.3 \quad (\#1\text{-parentheses}, \#4\text{-sub.}) \\
 &= 0.3[3.08] \div 0.2 - 0.72 \div 0.3 \quad (\#3\text{-multiplication}) \\
 &= 0.924 \div 0.2 - 0.72 \div 0.3 \quad (\#3\text{-division}) \\
 &= 4.62 - 0.72 \div 0.3 \quad (\#3\text{-division}) \\
 &= 4.62 - 2.4 \quad (\#4\text{-subtraction}) \\
 &= 2.22
 \end{aligned}$$

Ex. 4 $1 + 3^2 - (3.9 - 3.6 \div 3)^2 + (1.21)^1$

Solution:

$$\begin{aligned}
 &1 + 3^2 - (3.9 - 3.6 \div 3)^2 + (1.21)^1 \quad (\#1\text{-parentheses}, \#3\text{-div.}) \\
 &= 1 + 3^2 - (3.9 - 1.2)^2 + (1.21)^1 \quad (\#1\text{-parentheses}, \#4\text{-sub.}) \\
 &= 1 + 3^2 - (2.7)^2 + (1.21)^1 \quad (\#2\text{-exponents}) \\
 &= 1 + 3^2 - (2.7)^2 + (1.21)^1 \quad (\#2\text{-exponents}) \\
 &= 1 + 9 - (2.7)^2 + (1.21)^1 \quad (\#2\text{-exponents}) \\
 &= 1 + 9 - 7.29 + (1.21)^1 \quad (\#2\text{-exponents}) \\
 &= 1 + 9 - 7.29 + 1.21 \quad (\#4\text{-addition}) \\
 &= 10 - 7.29 + 1.21 \quad (\#4\text{-subtraction}) \\
 &= 2.71 + 1.21 \quad (\#4\text{-addition}) \\
 &= 3.92
 \end{aligned}$$

Objective b: Calculations with Decimals and Fractions.

Simplify the following:

Ex. 5 $(0.1)^3 \div \left\{3.1 - \frac{12}{5}\right\}^2 - \frac{1}{8} \div 0.5 + \frac{5}{16}$

Solution:

In this problem, we have both fractions and decimals. Let's see if the fractions convert to nice, terminating decimals:

$$\frac{12}{5} = 2.4 \qquad \frac{1}{8} = 0.125 \qquad \frac{5}{16} = 0.3125$$

Since we get nice, terminating decimals, we will do this as a decimal problem:

$$\begin{aligned}
& (0.1)^3 \div \left\{ \left(3.1 - \frac{12}{5} \right)^2 - \frac{1}{8} \div 0.5 \right\} + \frac{5}{16} \\
&= (0.1)^3 \div \left\{ \left(3.1 - 2.4 \right)^2 - 0.125 \div 0.5 \right\} + 0.3125 \\
&\quad \text{(#1-parentheses, #1-parentheses, #4-subtraction)} \\
&= (0.1)^3 \div \left\{ (0.7)^2 - 0.125 \div 0.5 \right\} + 0.3125 \quad \text{(#1-parent., #2-exp.)} \\
&= (0.1)^3 \div (0.49 - 0.125 \div 0.5) + 0.3125 \quad \text{(#1-parent., #3-division)} \\
&= (0.1)^3 \div (0.49 - 0.25) + 0.3125 \quad \text{(#1-parent., #4-subtraction)} \\
&= (0.1)^3 \div (0.24) + 0.3125 \quad \text{(#2-exponents)} \\
&= 0.001 \div (0.24) + 0.3125 \quad \text{(#3-division)} \\
&= 0.0041\overline{6} + 0.3125 \quad \text{(#4-addition)} \\
&= 0.31\overline{6}
\end{aligned}$$

Ex. 6 $(2.5)^2 - \left(\frac{5}{6} \div 0.75 \right) - \sqrt{0.09} \left(2\frac{5}{14} - 2.2 \right) \cdot 100$

Solution:

In this problem, we have both fractions and decimals. Let's see if the fractions convert to nice, terminating decimals:

$$\frac{5}{6} = 0.833333... \quad \frac{5}{14} = 0.35714285714...$$

Since we do not get nice, terminating decimals, we will do this as a fraction problem:

$$\begin{aligned}
2.5 &= 2\frac{5}{10} = 2\frac{1}{2}, \quad 0.75 = \frac{75}{100} = \frac{3}{4}, \quad \sqrt{0.09} = \sqrt{\frac{9}{100}}, \quad 2.2 = 2\frac{2}{10} = 2\frac{1}{5} \\
\text{So, } & (2.5)^2 - \left(\frac{5}{6} \div 0.75 \right) - \sqrt{0.09} \left(2\frac{5}{14} - 2.2 \right) \cdot 100 \\
&= \left(2\frac{1}{2} \right)^2 - \left(\frac{5}{6} \div \frac{3}{4} \right) - \sqrt{\frac{9}{100}} \left(2\frac{5}{14} - 2\frac{1}{5} \right) \cdot 100 \quad \text{(invert and multiply)} \\
&= \left(2\frac{1}{2} \right)^2 - \left(\frac{5}{6} \cdot \frac{4}{3} \right) - \sqrt{\frac{9}{100}} \left(2\frac{5}{14} - 2\frac{1}{5} \right) \cdot 100 \quad \text{(#1-parent., #3-mult.)} \\
&= \left(2\frac{1}{2} \right)^2 - \frac{10}{9} - \sqrt{\frac{9}{100}} \left(2\frac{5}{14} - 2\frac{1}{5} \right) \cdot 100 \quad \text{(L.C.D. = 70, build fractions)} \\
&= \left(2\frac{1}{2} \right)^2 - \frac{10}{9} - \sqrt{\frac{9}{100}} \left(2\frac{5 \cdot 5}{14 \cdot 5} - 2\frac{1 \cdot 14}{5 \cdot 14} \right) \cdot 100 \quad \text{(multiply)} \\
&= \left(2\frac{1}{2} \right)^2 - \frac{10}{9} - \sqrt{\frac{9}{100}} \left(2\frac{25}{70} - 2\frac{14}{70} \right) \cdot 100 \quad \text{(#1-parent., #4-sub.)} \\
&= \left(2\frac{1}{2} \right)^2 - \frac{10}{9} - \sqrt{\frac{9}{100}} \left(\frac{11}{70} \right) \cdot 100 \quad \text{(change into an improper fraction)} \\
&= \left(\frac{5}{2} \right)^2 - \frac{10}{9} - \sqrt{\frac{9}{100}} \left(\frac{11}{70} \right) \cdot 100 \quad \text{(#2-exponents)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{25}{4} - \frac{10}{9} - \frac{3}{10} \left(\frac{11}{70} \right) \cdot 100 && \text{(change into an improper fraction)} \\
&= \frac{25}{4} - \frac{10}{9} - \frac{3}{10} \left(\frac{11}{70} \right) \cdot \frac{100}{1} && \text{(#3-multiplication)} \\
&= \frac{25}{4} - \frac{10}{9} - \frac{33}{7} && \text{(L.C.D. = 252, build fraction)} \\
&= \frac{25 \cdot 63}{4 \cdot 63} - \frac{10 \cdot 28}{9 \cdot 28} - \frac{33 \cdot 36}{7 \cdot 36} && \text{(multiply)} \\
&= \frac{1575}{252} - \frac{280}{252} - \frac{1188}{252} && \text{(#4-subtraction)} \\
&= \frac{1295}{252} - \frac{1188}{252} && \text{(#4-subtraction)} \\
&= \frac{107}{252}
\end{aligned}$$

Since $252 = 4 \cdot 9 \cdot 7 = 2^2 \cdot 3^2 \cdot 7$ and 107 is not divisible by 2, 3, or 7, our answer is reduced to lowest terms. So, the answer is $\frac{107}{252}$.

Objective c: Applications

Solve the following:

Ex. 7 A developer purchased 409.84 acres of land and plans to divide it into a 52-acre recreational park and lots of $\frac{8}{11}$ of an acre each. If 12 acres will be reserved for roads and utilities, how many lots can be formed from the land?

Solution:

To begin, we must exclude the land that is being used for roads, utilities and the recreational park. The remaining land will be split into lots of $\frac{8}{11}$ acres each:

$$\begin{aligned}
&(409.84 - 52 - 12) \div \frac{8}{11} && \text{(#1-parenthesis, #4-subtraction)} \\
&= (345.84) \div \frac{8}{11} && \text{(write 345.62 as an improper fraction)} \\
&= \frac{34584}{100} \div \frac{8}{11} && \text{(invert and multiply)} \\
&= \frac{34584}{100} \cdot \frac{11}{8} && \text{(reduce)} \\
&= \frac{4323 \cdot 8}{100} \cdot \frac{11}{8 \cdot 1} = \frac{4323}{100} \cdot \frac{11}{1} && \text{(multiply and then divide)} \\
&= \frac{47553}{100} = 475.53
\end{aligned}$$

The developer can form 475 complete lots.

- Ex. 8 At the department store, Leroy purchased four boxes of cat food priced at \$5.78 per box, six t-shirts for \$4.95 each, and two pairs of shoes for \$19.48. If the sale tax was \$5.58, how much change did he get if he gave the cashier a hundred dollar bill?

Solution:

First, multiply the quantity times the price for each item he bought:

$$4(5.78) = 23.12 \qquad 6(4.95) = 29.70 \qquad 2(19.48) = 38.96$$

Add these amounts and the sales tax:

$$23.12 + 29.70 + 38.96 + 5.58 = 97.36$$

Now, subtract this from 100:

$$100 - 97.36 = 2.64$$

He received \$2.64 in change.

- Ex. 9 The monthly rainfall amounts for a six-month period of time were 1.34", 2.07", 4.56", 2.03", 0.29", and 1.34". Find the average rainfall per month (round the nearest hundredth).

Solution:

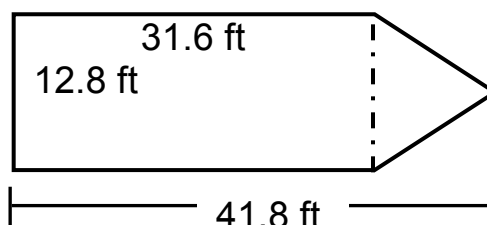
To find the average, we add the numbers and then divide by the

number of numbers: $(1.34 + 2.07 + 4.56 + 2.03 + 0.29 + 1.34) \div 6$

$$= 11.63 \div 6 = 1.938333... \approx 1.94"$$

The average rainfall was 1.94" per month.

- Ex. 10 Find the area of the following:

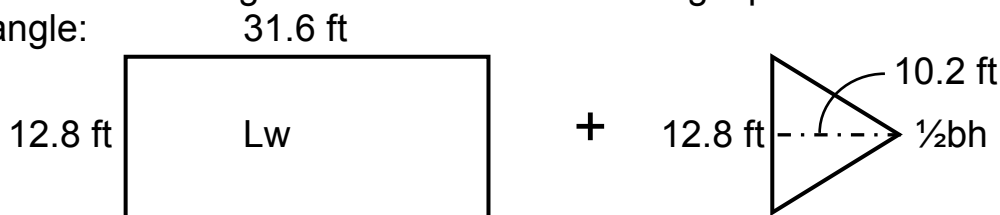


Solution:

Since the triangle is turned on its side, the height of the triangle is the total length of the figure minus the length of the rectangle:

$$41.8 \text{ ft} - 31.6 \text{ ft} = 10.2 \text{ ft}$$

The area of the figure is the area of a rectangle plus the area of the triangle:



$$A = 31.6(12.8) + \frac{1}{2}(12.8)(10.2) = 404.48 + 65.28 = 469.76$$

The area is 469.76 ft².