Sect 5.2 – Rates

Objective a: Definition of a Rate.

A **rate** is a comparison of two quantities that have different types of units. One unit cannot be converted into the other unit unlike ratios. We will express our rates using whole numbers. In our answer, since the units are not the same, we will leave our units in the answer. Also, the numbers in our answer will be reduced to lowest terms. Let's try some examples:

Write each rate in simplest form:

Ex. 1 $186 for 18 picture frames.

Solution:

$\frac{\$186}{18\,\text{frames}} = \frac{\$31}{3\,\text{frames}}$.

Ex. 2 Seventy-five miles on four gallons of gas.

Solution:

$\frac{75\,\text{miles}}{4\,\text{gal}}$.

Ex. 3 Fifty-six pizzas for 168 people.

Solution:

$\frac{56\,\text{pizzas}}{168\,\text{people}} = \frac{14\,\text{pizzas}}{42\,\text{people}} = \frac{2\,\text{pizzas}}{6\,\text{people}} = \frac{1\,\text{pizza}}{3\,\text{people}}$.

Objective b: Unit Rates.

A **Unit Rate** is a rate where the number in the denominator is one. To change a rate into a unit rate, divide the numerator by the denominator and write the result in the numerator.

Write as a unit rate:

Ex. 4 Seventy-five miles on four gallons of gas.

Solution:

$\frac{75\,\text{miles}}{4\,\text{gal}}$, now divide 75 by 4 to get 18.75,

So the unit rate is $\frac{18.75\,\text{miles}}{\text{gal}}$ or 18.75 miles per gallon.

Ex. 5 $3.36 for 16 ounces of cheese.

Solution:

$\frac{\$3.36}{16\text{oz}}$, now divide 3.36 by 16 to get 0.21. So, our unit rate is $\frac{\$0.21}{\text{oz}}$ or $0.21 per ounce.

Caution: Do not write $\frac{0.21¢}{\text{oz}}$ since 0.21¢ represent a fraction of a penny. If you want to use cents, write $\frac{21¢}{\text{oz}}$.

Ex. 6 98 pounds for 44 people

Solution:

$\frac{98\text{pounds}}{44\text{people}}$, now divide 98 by 44 to get $2.2\overline{27}$. So, our unit rate is $\frac{2.2\overline{27}\text{pounds}}{\text{person}}$ or $2.2\overline{27}$ pounds per person.

Objective c: Determining the best buy.

In shopping, it is important to get the most for your money. So, we try to find the item that is cheapest per unit. To do that, we write the cost over the number of units and convert that into a unit rate. Whichever answer is the smallest, that will be the best buy.

Find the best buy.

Ex. 7 $1.89 for a 32 oz jar of picante sauce.
$0.99 for a 18 oz jar of picante sauce.

Solution:

First, write each as a unit rate with the cost on top:

$\frac{\$1.89}{32\text{oz}} \approx \frac{\$0.059}{\text{oz}}$ and $\frac{\$0.99}{18\text{oz}} = \frac{\$0.055}{\text{oz}}$. Since $\frac{\$0.055}{\text{oz}}$ is cheaper, then the 18 oz jar is the better buy.

Ex. 8 $2.39 for 20 grams of all spice seasoning.
$0.99 for 8 grams of all spice seasoning.

Solution:

First, write each as a unit rate with the cost on top:

$\frac{\$2.39}{20\text{grams}} = \frac{\$0.1195}{\text{gram}}$ and $\frac{\$0.99}{8\text{grams}} = \frac{\$0.12375}{\text{gram}}$. Since $\frac{\$0.1195}{\text{gram}}$ is cheaper, then the 20 grams of all spice seasoning is the better buy.

Objective d: Applications

Ex. 9 Juan is trying to decide whether to buy a 20-ounce box of Cheeros for \$3.29 or to buy a 15-ounce box of Hill Country Fare Toasted Oats for \$1.99.

a) Which is the better buy?

b) If he has a dollar off coupon on Cheeros, will that change which one is the better buy?

Solution:

a) First, write each as a unit rate with the cost on top: $\frac{\$3.29}{20\,\text{oz}} = \frac{\$0.1645}{\text{oz}}$ and $\frac{\$1.99}{15\,\text{oz}} \approx \frac{\$0.1327}{\text{oz}}$. Since $\frac{\$0.1327}{\text{oz}}$ is cheaper, then the Hill Country Fare Toasted Oats is the better buy.

b) With a dollar off coupon for Cheeros, the 20 oz box of Cheeros will now cost \$2.29. So, the unit rate for Cheeros is now equal to $\frac{\$2.29}{20\,\text{oz}} = \frac{\$0.1145}{\text{oz}}$ which is cheaper than $\frac{\$0.1327}{\text{oz}}$. So yes, the coupon does make the Cheeros the better buy.

Ex. 10 A machine can produce 12 tapered pins in 40 seconds. A second machine can produce 43 tapered pins in 150 seconds. Which machine is faster?

Solution:

We will first compute the unit rate for each machine and then compare:

1st machine: $\frac{12\,\text{pins}}{40\,\text{sec}}$ = 0.3 pins per second

2nd machine: $\frac{43\,\text{pins}}{150\,\text{sec}}$ = 0.2866… pins per second.

The first machine is faster.