Sect 5.3 – Proportions

Objective a: Definition of a Proportion.

A **proportion** is a statement that two ratios or two rates are equal.

A proportion takes a form that is similar to an analogy in English class. For example, a puppy is to a dog as a kitten is to a cat is an analogy, but we can write this as a proportion. A puppy is to a dog would be a fraction on one side of the equal sign and a kitten is to a cat would be a fraction on the other side:

$$\frac{\text{puppy}}{\text{dog}} = \frac{\text{kitten}}{\text{cat}}$$

Write the following as a proportion:

Ex. 1 Fifty feet of fencing is to 35 pounds as thirty feet is to 21 pounds.

Solution:

Fifty feet of fencing is to 35 pounds : $\frac{50\,\text{ft}}{35\,\text{pounds}}$

thirty feet is to 21 pounds: $\frac{30\,\text{ft}}{21\,\text{pounds}}$

So, the proportion is $\frac{50\,\text{ft}}{35\,\text{pounds}} = \frac{30\,\text{ft}}{21\,\text{pounds}}$.

Ex. 2 \$7 is to 2.1 liters as \$2.40 is to 0.72 liters.

Solution:

\$7 is to 2.1 liters: $\frac{\$7}{2.1\text{ liters}}$; \$2.40 is to 0.72 liters: $\frac{\$2.40}{0.72\text{ liters}}$

So, the proportion is $\frac{\$7}{2.1\text{ liters}} = \frac{\$2.40}{0.72\text{ liters}}$.

Ex. 3 $1\frac{4}{5}$ is to $5\frac{1}{7}$ as $\frac{1}{6}$ is to $\frac{10}{21}$.

Solution:

So, the proportion is $\dfrac{1\frac{4}{5}}{5\frac{1}{7}} = \dfrac{\frac{1}{6}}{\frac{10}{21}}$.

Objective a2: Introduction to equations and solving equations using the multiplication property of equality.

In example #1, it is fairly easy to see that the proportion is true since $\frac{50\text{ ft}}{35\text{ pounds}}$ reduces to $\frac{10\text{ ft}}{7\text{ pounds}}$ and $\frac{30\text{ ft}}{21\text{ pounds}}$ also reduces to $\frac{10\text{ ft}}{7\text{ pounds}}$. In example #2 and #3 however, it is much more difficult to check to see if the proportion is true. We need to find a more efficient way to check to see if a proportion is true. To help us do this, we need introduce the idea of equations and how to solve equations using the multiplication property of equality.

In Algebra, when there is a number that we do not know its value, we represent the number using a letter like x. So, if we want to write five times an unknown number, we write 5•x or 5x. Using this idea, we can use what are called equations to help us find these unknown numbers for a particular example. Let's state a definition.

An **equation** is a statement that two quantities are equal. An equation can be as simple as 4 quarters = \$1, or it could be more complex like $3x = 2.7$ and $2x^2 – 5x + 4 = 23$. A **solution** to an equation is the value of x that makes the equation true. For example, $x = 0.9$ is a solution to the equation $3x = 2.7$ since if we replace x by 0.9 and do the multiplication, we get: $3(0.9) = 2.7$.

Solve the following:

Ex. 4 $4x = 28$

Solution:

Since $4 \bullet 7 = 28$, then $x = 7$ has to be the solution. We can also get the solution by dividing 28 by 4 since $28 \div 4 = 7$.

The property the allows us to solve this equation is called the multiplication property of equality. It says we can multiply or divide both sides by the same non-zero number. In our example, we are dividing both sides by 4 to solve for x:

Multiplication Property of Equality:

If A = B and C ≠ 0, then A•C = B•C and $\frac{A}{C} = \frac{B}{C}$ are equivalent equations to A = B (i.e., they have the same solutions)

Solve the following:

Ex. 5a $2.4x = 9.84$

Solution:
We will divide both sides by the number in front of x:

$$\frac{2.4x}{2.4} = \frac{9.84}{2.4}$$

$$x = 9.84 \div (2.4)$$

$$x = 4.1$$

Ex. 5b $\frac{3}{4}x = \frac{5}{6}$

Solution:
We will divide both sides by the number in front of x:

$$\frac{\frac{3}{4}x}{\frac{3}{4}} = \frac{\frac{5}{6}}{\frac{3}{4}}$$

$$x = \frac{5}{6} \div \frac{3}{4}$$

$$x = \frac{5}{6} \bullet \frac{4}{3} = \frac{10}{9}$$

Objective b: Using cross multiplication to determine if a proportion is true.

With a proportion like $\frac{a}{b} = \frac{c}{d}$, if we multiply both sides by b•d, we get:

$$b \bullet d \bullet \frac{a}{b} = b \bullet d \bullet \frac{c}{d} \qquad \text{(write b and d over 1)}$$

$$\frac{b}{1} \bullet \frac{d}{1} \bullet \frac{a}{b} = \frac{b}{1} \bullet \frac{d}{1} \bullet \frac{c}{d} \qquad \text{(reduce)}$$

$$\frac{1}{1} \bullet \frac{d}{1} \bullet \frac{a}{1} = \frac{b}{1} \bullet \frac{1}{1} \bullet \frac{c}{1} \qquad \text{(simplify)}$$

$$ad = bc$$

This says that $\frac{a}{b} = \frac{c}{d}$ if and only if ad = bc (provided that neither b nor d are 0). This gives use an easy way to check to see if a proportion is true by multiplying the first numerator with the second denominator and seeing if it is equal to the product of the second numerator with the first denominator. This technique is called cross multiplication:

Cross Multiplication

For $b \neq 0$ and $d \neq 0$, $\frac{a}{b} = \frac{c}{d}$ if and only if ad = bc. “ad” and “bc” are called the cross products.

Now, let’s look back at the results from first three examples and use cross multiplication to show that the proportions are true.

Use cross multiplication to determine if the following proportions are true:

Ex. 6 $\frac{50\text{ ft}}{35\text{ pounds}} = \frac{30\text{ ft}}{21\text{ pounds}}$

Solution:

$\frac{\boxed{50\text{ ft}}}{35\text{ pounds}} = \frac{30\text{ ft}}{\boxed{21\text{ pounds}}}$ (cross multiply)

$50 \bullet 21 = 35 \bullet 30$

$1050 = 1050$ True

Ex. 7 $\frac{\$7}{2.1\text{ liters}} = \frac{\$2.40}{0.72\text{ liters}}$

Solution:

$\frac{\boxed{\$7}}{2.1\text{ liters}} = \frac{\$2.40}{\boxed{0.72\text{ liters}}}$ (cross multiply)

$7 \bullet 0.72 = 2.1 \bullet 2.40$

$5.04 = 5.04$ True

Ex. 8 $\frac{1\frac{4}{5}}{5\frac{1}{7}} = \frac{\frac{1}{6}}{\frac{10}{21}}$

Solution:

$\frac{1\frac{4}{5}}{\boxed{5\frac{1}{7}}} = \frac{\boxed{\frac{1}{6}}}{\frac{10}{21}}$ (cross multiply)

$1\frac{4}{5} \bullet \frac{10}{21} = 5\frac{1}{7} \bullet \left(\frac{1}{6}\right)$ (change to improper fractions)

$\frac{9}{5} \bullet \frac{10}{21} = \frac{36}{7} \bullet \frac{1}{6}$ (reduce)

$\frac{3}{1} \bullet \frac{2}{7} = \frac{6}{7} \bullet \frac{1}{1}$ (simplify)

$\frac{6}{7} = \frac{6}{7}$ True

Ex. 9 $\frac{5.37}{8.5} = \frac{11.63}{18.6}$

Solution:

$\frac{5.37}{\boxed{8.5}} = \frac{\boxed{11.63}}{18.6}$

5.37•18.6 = 8.5•11.63

99.882 = 98.855 False

Objective c: Solving proportions with an unknown number.

Now, we will examine how to solve for a missing number in a proportion. First, we will cross multiply. Next, we will simplify each cross product. Finally, we will use the multiplication property of equality to solve for the unknown number. Let's try some examples.

Solve:

Ex. 10 $\frac{n}{8} = \frac{7}{11}$

Solution:

$\frac{n}{8} = \frac{7}{11}$ (cross multiply)

n•11 = 8•7 (simplify)

11n = 56 (divide by 11)

$\frac{11n}{11} = \frac{56}{11}$

n = $\frac{56}{11}$ or $5\frac{1}{11}$ or $5.\overline{09}$.

Ex. 11 $\frac{0.8}{11} = \frac{9.5}{n}$

Solution:

$\frac{0.8}{11} = \frac{9.5}{n}$ (cross multiply)

0.8•n = 11•9.5 (simplify)

0.8n = 104.5 (divide by 0.8)

$\frac{0.8n}{0.8} = \frac{104.5}{0.8}$

n = 104.5 ÷ 0.8 = 130.625.

Ex. 12 $\frac{7.8}{n} = \frac{5.1}{9}$

Solution:

$\frac{7.8}{n} = \frac{5.1}{9}$ (cross multiply)

7.8•9 = n•5.1 (simplify)

70.2 = 5.1n (divide by 5.1)

$\frac{70.2}{5.1} = \frac{5.1n}{5.1}$ Since 70.2 ÷ 5.1 is a long messy decimal, let's express the answer as a fraction:

n = $\frac{70.2}{5.1} = \frac{702}{51} = 13\frac{39}{51}$ (move the decimal and reduce)

n = $13\frac{13}{17}$.

Ex. 13 $\dfrac{3\frac{1}{4}}{6\frac{3}{11}} = \dfrac{n}{5\frac{4}{13}}$

<u>Solution:</u>

$\dfrac{3\frac{1}{4}}{6\frac{3}{11}} = \dfrac{n}{5\frac{4}{13}}$ (cross multiply)

$\left(3\frac{1}{4}\right)\left(5\frac{4}{13}\right) = \left(6\frac{3}{11}\right)n$ (change to improper fractions)

$\frac{13}{4}\bullet\frac{69}{13} = \frac{69}{11}\bullet n$ (reduce)

$\frac{1}{4}\bullet\frac{69}{1} = \frac{69}{11}\bullet n$ (simplify)

$\frac{69}{4} = \frac{69}{11}n$ (divide by $\frac{69}{11}$)

$$\frac{\frac{69}{4}}{\frac{69}{11}} = \frac{\frac{69}{11}n}{\frac{69}{11}}$$

$n = \frac{69}{4} \div \frac{69}{11}$ (invert and multiply)

$= \frac{69}{4}\bullet\frac{11}{69}$ (reduce)

$= \frac{1}{4}\bullet\frac{11}{1}$ (simplify)

$= \frac{11}{4}$ or $2\frac{3}{4}$ or 2.75