Sect 5.4 – Applications of Proportions

Objective a: Applications of Proportions.

Solve the following using a proportion:

Ex. 1 In order for a certain engine to run efficiently, the air to fuel ratio needs to be 14.7 to 1. If an engine used 12 pounds of fuel during a test run, how many pounds of air did it draw if it ran efficiently?

Solution:

Let A = the number of pounds of air.

Let's write the pounds of air on top and the pounds of fuel on the bottom so, " the air to fuel ratio needs to be 14.7 to 1: $\frac{14.7}{1}$

The engine used 12 pounds, so that number would go on the bottom: $\frac{A}{12}$

Thus, the proportion is: $\frac{14.7}{1} = \frac{A}{12}$ (cross multiply)

$14.7 \bullet 12 = 1 \bullet A$ (simplify)

$176.4 = A$

Thus, the engine draws 176.4 pounds of air during the test run.

Ex. 2 If one twelve-ounce can of soda contains 0.25 cups of sugar, how much sugar (to the nearest hundredth) does a twenty-ounce bottle of soda contain?

Solution:

Let S = the amount of sugar in a 20 oz bottle of soda.

Let's write soda over sugar so a "twelve-ounce can of soda contains 0.25 cups of sugar": $\frac{12}{0.25}$. We have a 20 oz bottle, so 20 will go on top: $\frac{20}{S}$. Thus, the proportion is: $\frac{12}{0.25} = \frac{20}{S}$ (cross multiply)

$12 \bullet S = 0.25 \bullet 20$

$12S = 5$ (divide by 12)

$\frac{12S}{12} = \frac{5}{12}$

$S = 0.416... \approx 0.42$ cups of sugar.

Ex. 3 The dosage for a certain medication is five drops for every eight pounds of body weight. If a child weighs fifty-four pounds, how many drops of medication should he receive?

Solution:

Let m = the amount of medication for a child weighing 54 lb.

Let's write medication over weight:

$\frac{\text{Med.}}{\text{Weight}}$: $\frac{5}{8} = \frac{m}{54}$ (cross multiply)

$5 \bullet 54 = 8 \bullet m$ (simplify)

$270 = 8m$ (divide by 8)

$\frac{270}{8} = \frac{8m}{8}$

$m = 33.75 \approx 34$ drops

Ex. 4 At a particular college, there are 15 female students to every 13 male students. If there is 35,000 students enrolled at the college, how many of the students are female?

Solution:

Since there are 15 female students to every 13 male students, then there are 15 female students to every 15 + 13 = 28 students.

Let F = the number of female students at the college

Let's write the number of female over the total:

$\frac{\text{Females}}{\text{Total}}$: $\frac{15}{28} = \frac{F}{35000}$ (cross multiply)

$15 \bullet 35000 = 28 \bullet F$ (simplify)

$525000 = 28F$ (divide by 28)

$\frac{525000}{28} = \frac{28F}{28}$

$F = 18{,}750$ females

Ex. 5 On a blueprint, 3 inches corresponds to 4 feet in real life. If a wall measures 10.5 inches on the blueprint, how long is it in real life?

Solution:

Let L = the length of the wall in real life.

Let's write blueprint over real life:

$\frac{\text{blueprint}}{\text{real life}}$: $\frac{3}{4} = \frac{10.5}{L}$ (cross multiply)

$3 \bullet L = 4 \bullet 10.5$ (simplify)

$3L = 42$ (divide by 3)

$\frac{3L}{3} = \frac{42}{3}$ = 14 feet in real life.

Ex. 6 In a test of 180 steering wheels, 7 were found to be defective. If 5580 steering wheels were made, how many would you expect to be defective?

<u>Solution:</u>

Let d = the number that are defective out of 5580 steering wheels.

Let's write the total over the number defective:

$$\frac{\text{total}}{\text{defective}}: \quad \frac{180}{7} = \frac{5580}{d} \qquad \text{(cross multiply)}$$

$$180 \bullet d = 7 \bullet 5580 \quad \text{(simplify)}$$

$$180d = 39{,}060 \quad \text{(divide by 180)}$$

$$\frac{180d}{180} = \frac{39060}{180}$$

$$d = 217$$

There were 217 defective steering wheels.

Ex. 7 If $\frac{2}{3}$ of an inch represents 70 miles on a map, how far apart are two cities that are $6\frac{5}{12}$ inches apart on the map?

<u>Solution:</u>

Let d = the distance in real life.

Let's write the distance on the map over the distance in real life:

$$\frac{\text{map}}{\text{real life}}: \quad \frac{\frac{2}{3}}{70} = \frac{6\frac{5}{12}}{d} \quad \text{(cross multiply)}$$

$$\frac{2}{3} \bullet d = 70 \bullet 6\frac{5}{12} \quad \text{(change into improper fractions)}$$

Since $6\frac{5}{12} = \frac{77}{12}$, then the problem becomes:

$$\frac{2}{3} \bullet d = \frac{70}{1} \bullet \frac{77}{12} \quad \text{(simplify)}$$

$$\frac{2}{3}d = \frac{35}{1} \bullet \frac{77}{6}$$

$$\frac{2}{3}d = \frac{2695}{6} \quad \left(\text{divide by } \frac{2}{3}\right)$$

$$\frac{\frac{2}{3}d}{\frac{2}{3}} = \frac{\frac{2695}{6}}{\frac{2}{3}}$$

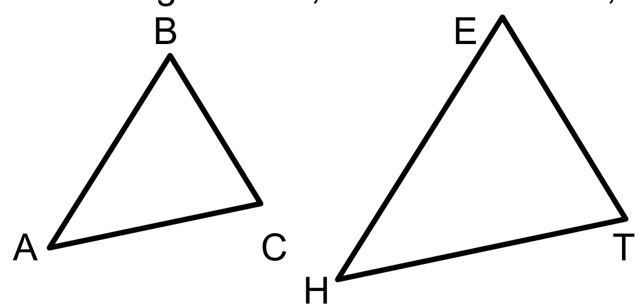
$$d = \frac{2695}{6} \div \frac{2}{3} = \frac{2695}{6} \bullet \frac{3}{2} = \frac{2695}{2} \bullet \frac{1}{2} = \frac{2695}{4} = 673.75$$

So, the two cities are 673.75 miles apart.

Objective b: Similar Triangles

Two triangles are **similar** if they have the same shape, but not necessarily the same size. The notation for writing that triangle ABC is similar to triangle HET is **ΔABC ~ ΔHET**. The ordering of the letters shows the corresponding vertices. In this case, $\angle$A corresponds to $\angle$H, $\angle$B corresponds to $\angle$E and $\angle$C corresponds to $\angle$T. In similar triangles, the corresponding angles are equal. Thus, if ΔABC ~ ΔHET, then m$\angle$A = m$\angle$H, m$\angle$B = m$\angle$E, and m$\angle$C = m$\angle$T. The corresponding sides are not equal. However, the ratios of corresponding sides are equal since one triangle is in proportion to the other triangle. Thus, if ΔABC ~ ΔHET, then:

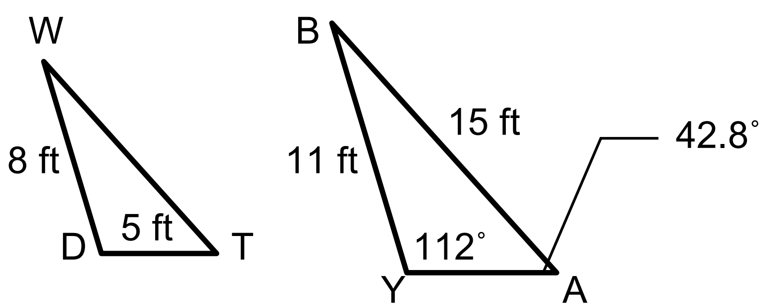
$$\frac{AB}{HE} = \frac{BC}{ET} = \frac{AC}{HT}$$



We can use proportions to then find the missing sides in a pair of similar triangles. In setting up a proportion, we always start with a pair of corresponding sides that of which we know both values. Let's try some examples.

Find the missing sides and angles of the following:

Ex. 8 ΔDWT ~ ΔYBA



Solution:

The pair of corresponding sides we will start with are DW and YB. We write the length from the smaller triangle over the length from the bigger triangle. So, our proportions are:

$\frac{DW}{BY} = \frac{WT}{BA}$ and $\frac{DW}{BY} = \frac{DT}{YA}$

$\frac{8}{11} = \frac{WT}{15}$ and $\frac{8}{11} = \frac{5}{YA}$. Now, cross multiply and solve:

$\frac{8}{11} = \frac{WT}{15}$	$\frac{8}{11} = \frac{5}{YA}$.
11WT = 8•15	8YA = 11•5

$$\frac{11WT}{11} = \frac{120}{11} \qquad\qquad \frac{8YA}{8} = \frac{55}{8}$$

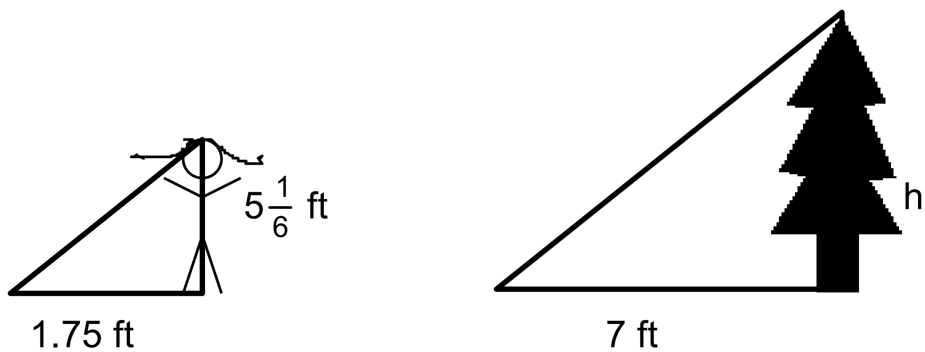
$$WT = 10\frac{10}{11} \text{ ft} \qquad\qquad YA = 6.875 \text{ ft.}$$

Since $m\angle D = m\angle Y$ and $m\angle T = m\angle A$, then $m\angle D = 112^\circ$ and $m\angle T = 42.8^\circ$. Also, $m\angle W = m\angle B = 180^\circ - 112^\circ - 42.8^\circ = 25.2^\circ$.

Ex. 9 At 4 p.m., Juanita's shadow was 1.75 feet long while a tree's shadow was 7 feet long. If Juanita is $5\frac{1}{6}$ feet tall, how tall is the tree?

Solution:

We begin by drawing a picture:



Let h = the height of the tree.

Let's write the height over the length of the shadow:

$$\frac{\text{Juanita}}{\text{Tree}}: \qquad \frac{5\frac{1}{6}}{h} = \frac{1.75}{7} \text{ (cross multiply)}$$

$$5\frac{1}{6} \bullet 7 = h \bullet 1.75 \quad \text{(change into improper fractions)}$$

$5\frac{1}{6} = \frac{31}{6}$ and $1.75 = 1\frac{75}{100} = 1\frac{3}{4} = \frac{7}{4}$. So, the proportion becomes:

$$\frac{31}{6} \bullet \frac{7}{1} = \frac{7}{4} \bullet h \qquad \text{(simplify)}$$

$$\frac{217}{6} = \frac{7}{4}h \text{ (divide by } \frac{7}{4}\text{)}$$

$$\frac{\frac{217}{6}}{\frac{7}{4}} = \frac{\frac{7}{4}h}{\frac{7}{4}}$$

$$h = \frac{217}{6} \div \frac{7}{4} = \frac{217}{6} \bullet \frac{4}{7} = \frac{31}{3} \bullet \frac{2}{1} = \frac{62}{3} = 20\frac{2}{3}$$

So, the tree is $20\frac{2}{3}$ feet tall.