Sect 5.7 - Division of Polynomials

Concept #1: Division by a monomial.

We can use the same techniques and rules established in Sect 5.1 – 5.3 to simplify a monomial divided by a monomial.

Simplify the following. Write your answer using positive exponents:

Ex. 1 $\frac{-27a^7b^2}{45a^2b^{11}}$

Solution:

First, reduce the numbers by 9: $\frac{-27}{45} = -\frac{3}{5}$

When dividing powers of the same base, subtract the exponents. Thus, for the factors of a, the exponent of a will become 7 – 2 = 5. Since there were more factors of a in the numerator, then a^5 will be in the numerator. For the factors of b, the exponent of b will become 11 – 2 = 9. Since there were more factors of b in the denominator, then b^9 will be in the denominator. Hence,

$$\frac{-27a^7b^2}{45a^2b^{11}} = -\frac{3a^5}{5b^9}$$

Ex. 2 $\frac{-75x^5(y^2)^4}{-175x^2y^3x^4}$

Solution:

First, simplify the numerator and the denominator:

Since $(y^2)^4 = y^8$ and $x^2 \bullet x^4 = x^6$, then $\frac{-75x^5(y^2)^4}{-175x^2y^3x^4} = \frac{-75x^5y^8}{-175x^6y^3}$

Next, reduce the numbers by 25: $\frac{-75}{-175} = \frac{3}{7}$

When dividing powers of the same base, subtract the exponents. Thus, for the factors of x, the exponent of x will become 6 – 5 = 1. Since there were more factors of x in the denominator, then x will be in the denominator. For the factors of y, the exponent of y will become 8 – 3 = 5. Since there were more factors of y in the numerator, then y^5 will be in the numerator. Hence,

$$\frac{-75x^5y^8}{-175x^6y^3} = \frac{3y^5}{7x}$$

To see how a polynomial divided a monomial works, let us consider the following example:

Simplify:

Ex. 3 $\frac{3x^2-6x+12}{3}$

Solution:

Since $\frac{3x^2-6x+12}{3}$ is the same as $\frac{1}{3}(3x^2 - 6x + 12)$, we can distribute the $\frac{1}{3}$ to each term and simplify:

$$\frac{1}{3}(3x^2) - \frac{1}{3}(6x) + \frac{1}{3}(12) = \frac{3x^2}{3} - \frac{6x}{3} + \frac{12}{3} = x^2 - 2x + 4.$$

Notice that we divided each term by 3 to get the answer.

In general, to divide a polynomial by a monomial, divide each term of the polynomial by the monomial and simplify.

Simplify:

Ex. 4 $\frac{25t^3-15t^2+30t}{15t}$

Solution:

$$\frac{25t^3-15t^2+30t}{15t} = \frac{25t^3}{15t} - \frac{15t^2}{15t} + \frac{30t}{15t} = \frac{5}{3}t^2 - t + 2$$

Ex. 5 $(18x^6 - 27x^5 + 3x^3) \div (3x^3)$

Solution:

$$(18x^6 - 27x^5 + 3x^3) \div (3x^3) = \frac{18x^6}{3x^3} - \frac{27x^5}{3x^3} + \frac{3x^3}{3x^3} = 6x^3 - 9x^2 + 1$$

Ex. 6 $\frac{8xy^2-16x^2y+24xy}{4x^2y^2}$

Solution:

$$\frac{8xy^2-16x^2y+24xy}{4x^2y^2} = \frac{8xy^2}{4x^2y^2} - \frac{16x^2y}{4x^2y^2} + \frac{24xy}{4x^2y^2} = \frac{2}{x} - \frac{4}{y} + \frac{6}{xy}$$

Ex. 7 $(42a^6b^5 - 3a^5b^7 + 21a^3b^8) \div (7a^5b^7)$

Solution:

$$(42a^6b^5 - 3a^5b^7 + 21a^3b^8) \div (7a^5b^7) = \frac{42a^6b^5}{7a^5b^7} - \frac{3a^5b^7}{7a^5b^7} + \frac{21a^3b^8}{7a^5b^7}$$

$$= \frac{6a}{b^2} - \frac{3}{7} + \frac{3b}{a^2}$$

Concept #2 Long Division

The process of dividing a polynomial by a polynomial is similar to performing long division with numbers. Let's look at a long division problem:

Simplify:

Ex. 8 $5297 \div 22$

Solution:

22 goes into 52 twice. Subtract 22•2 = 44 from 52 to get 8. Bring down the 9. 22 goes into 89 four times. Subtract 22•4 = 88 from 89 to get 1. Bring down the 7. Write the remainder over the divisor.

$$\begin{array}{r|l} & 240 + \frac{17}{22} \\ 22 & 5297 \\ & -44 \\ & \;\;89 \\ & -88 \\ & \;\;\;17 \end{array}$$

The major difference with polynomials is that instead of determining how many times 22 goes into 52, we take the first term in the dividend and divide it by the first term of the divisor. We use that result to continue the process. Let's see how this works.

Simplify:

Ex. 9 $(5x^2 + 6x + 3) \div (x + 2)$

Solution:

$\frac{5x^2}{x} = 5x$, subtract $5x(x + 2)$ which is the same as adding $-5x^2 - 10x$.

Bring down the 3. $\frac{-4x}{x} = -4$, subtract $-4(x + 2)$ or add $4x + 8$.

Put the remainder over the divisor.

So, the answer is $5x - 4 + \frac{11}{x+2}$.

$$\begin{array}{r|l} & 5x - 4 + \frac{11}{x+2} \\ x + 2 & 5x^2 + 6x + 3 \\ & -5x^2 - 10x \\ & \qquad\; -4x + 3 \\ & \qquad\; +4x + 8 \\ & \qquad\qquad\;\; 11 \end{array}$$

Long division is only used when dividing by a polynomial with more than one term. It is extremely important that the polynomials are written in order of descending powers before you begin the division process. Also, if there are some terms missing in the dividend, be sure to leave some space for where that term would have gone. Lastly, be sure to line up like terms.

Ex. 10 $\frac{-13+3x^2-2x}{x-2}$

Solution:

First, write the polynomials in order of descending powers:

$\frac{-13+3x^2-2x}{x-2} = \frac{3x^2-2x-13}{x-2}$

$\frac{3x^2}{x}$ = 3x, subtract 3x(x – 2) which is the same as adding $-3x^2 + 6x$.

Bring down the – 13. $\frac{4x}{x}$ = 4,

subtract 4(x – 2) or add – 4x + 8.

Put the remainder over the divisor.

So, the answer is $3x + 4 - \frac{5}{x-2}$.

$$
\begin{array}{r|l}
 & 3x + 4 - \frac{5}{x-2} \\
\cline{2-2}
x - 2 & 3x^2 - 2x - 13 \\
 & \underline{-3x^2 + 6x} \\
 & \quad 4x - 13 \\
 & \quad \underline{-4x + 8} \\
 & \qquad -5
\end{array}
$$

Ex. 11 $(25x^2 - 16) \div (5x + 4)$

Solution:

There is no x term in the dividend, so we will need to leave some space for it: $(25x^2 \qquad - 16) \div (5x + 4)$

$\frac{25x^2}{5x}$ = 5x, subtract 5x(5x + 4) which is the same as adding $-25x^2 - 20x$.

Bring down the – 16. $\frac{-20x}{5x}$ = – 4,

subtract – 4(5x + 4) or add 20x + 16.

There is no remainder.

So, the answer is 5x – 4.

$$
\begin{array}{r|l}
 & 5x - 4 \\
\cline{2-2}
5x + 4 & 25x^2 \qquad - 16 \\
 & \underline{-25x^2 - 20x} \\
 & \qquad -20x - 16 \\
 & \qquad \underline{20x + 16} \\
 & \qquad\qquad 0
\end{array}
$$

Ex. 12 $(3x^2 + 5x - 4) \div (5x)$

Solution:

Since we are dividing by a monomial, we do not use long division. We divide term by term:

$(3x^2 + 5x - 4) \div (5x)$

$= \frac{3x^2}{5x} + \frac{5x}{5x} - \frac{4}{5x}$ (reduce)

$= \frac{3}{5}x + 1 - \frac{4}{5x}$

Ex. 13 $(3x^2 + 5x - 4) \div (5 + x)$

Solution:

Since we are not dividing by a monomial, we must use long division:
First, write the polynomials in order of descending powers:

$\frac{3x^2+5x-4}{5+x} = \frac{3x^2+5x-4}{x+5}$

$\frac{3x^2}{x}$ = 3x, subtract 3x(x + 5) which is the same as adding $-3x^2 - 15x$.
Bring down the – 4. $\frac{-10x}{x} = -10$,
subtract – 10(x + 5) or add 10x + 50.
Put the remainder over the divisor.
So, the answer is $3x - 10 + \frac{46}{x+5}$.

$$
\begin{array}{r|l}
 & 3x - 10 + \frac{46}{x+5} \\
x + 5 & 3x^2 + 5x - 4 \\
 & \underline{-3x^2 - 15x} \\
 & -10x - 4 \\
 & \underline{10x + 50} \\
 & 46
\end{array}
$$

Ex. 14 $(8x^3 + 1) \div (4x^2 - 2x + 1)$

Solution:

There is some middle terms missing in the dividend, so we will need to leave some space for them:

$\frac{8x^3}{4x^2}$ = 2x, subtract $2x(4x^2 - 2x + 1)$ or add $-8x^3 + 4x^2 - 2x$. Bring down the 1. $\frac{4x^2}{4x^2}$ = 1, subtract $1(4x^2 - 2x + 1)$ or add $-4x^2 + 2x - 1$.
There is no remainder.
So, the answer is 2x + 1.

$$
\begin{array}{r|l}
 & 2x + 1 \\
4x^2 - 2x + 1 & 8x^3 \qquad\qquad + 1 \\
 & \underline{-8x^3 + 4x^2 - 2x} \\
 & 4x^2 - 2x + 1 \\
 & \underline{-4x^2 + 2x - 1} \\
 & 0
\end{array}
$$

Ex. 15 $(8x^3 + 1) \div (4x^2)$

Solution:

Since we are dividing by a monomial, we do not use long division.
We divide term by term:

$(8x^3 + 1) \div (4x^2)$

$= \frac{8x^3}{4x^2} + \frac{1}{4x^2}$ (reduce)

$= 2x + \frac{1}{4x^2}$

Ex. 16 $\frac{-9x^2-3+11x+2x^3}{2x-3}$

Solution:

First, write the polynomials in order of descending powers:

$\frac{-9x^2-3+11x+2x^3}{2x-3} = \frac{2x^3-9x^2+11x-3}{2x-3}$

$\frac{2x^3}{2x} = x^2$, subtract $x^2(2x - 3)$ or adding $-2x^3 + 3x^2$. Bring down the $11x$. $\frac{-6x^2}{2x} = -3x$, subtract $-3x(2x - 3)$ or add $6x^2 - 9x$. Bring down the -3. $\frac{2x}{2x} = 1$, subtract $1(2x - 3)$ or add $-2x + 3$. There is no remainder, so the answer is $x^2 - 3x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 3x + 1 \\
\hline
2x - 3 & 2x^3 - 9x^2 + 11x - 3 \\
 & \underline{-2x^3 + 3x^2} \\
 & -6x^2 + 11x \\
 & \underline{6x^2 - 9x} \\
 & 2x - 3 \\
 & \underline{-2x + 3} \\
 & 0
\end{array}
$$

Ex. 17 $(x^4 + 5x^3 + 4x^2 - 6) \div (x^2 + 5)$

Solution:

$\frac{x^4}{x^2} = x^2$, subtract $x^2(x^2 + 5)$ or add $-x^4 - 5x^2$, being careful to line up like terms. Bring down the -6. $\frac{5x^3}{x^2} = 5x$, subtract $5x(x^2 + 5)$ or add $-5x^3 - 25x$, being careful to line up like terms. $\frac{-x^2}{x^2} = -1$, subtract $-1(x^2 + 5)$ or add $x^2 + 5$. Since the degree of the divisor is higher than what is left, we stop and put the remainder over the divisor.

$$
\begin{array}{r|l}
 & x^2 + 5x - 1 + \frac{-25x-1}{x^2+5} \\
\hline
x^2 + 5 & x^4 + 5x^3 + 4x^2 \qquad - 6 \\
 & \underline{-x^4 \qquad - 5x^2} \\
 & 5x^3 - x^2 \qquad - 6 \\
 & \underline{-5x^3 \qquad - 25x} \\
 & -x^2 - 25x - 6 \\
 & \underline{x^2 \qquad + 5} \\
 & -25x - 1
\end{array}
$$