# Sect 6.1 & 6.2 – Understanding Percents

The word percent means "out of 100" or "per 100". We use the symbol "%" to denote a percent. Thus, we can write fifteen percent as 15%. In the 2004 election, when we say that John Kerry received 48% of the popular vote, that means that 48 out of every 100 voters voted for John Kerry. Likewise, since George Bush received 51% of the popular vote, that means that 51 out of every 100 voters voted for George Bush. We can also think of a percent of the numerator of fraction with denominator of 100. In other words, if 48 out of 100 people voted for John Kerry, then the fraction of the people that voted for John Kerry is $\frac{48}{100}$. Since the numerator is 48, then 48% voted for John Kerry. Let's consider the follow example:

Consider the following figure and answer the questions:

Ex. 1
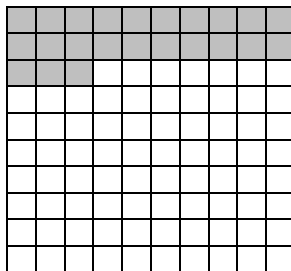
a)   What percent of the figure is shaded?
b)   What percent of the figure is not shaded?
Solution:
a)   Since $\frac{23}{100}$ of the figure is shaded, then 23% is shaded.

b)   Since $\frac{77}{100}$ of the figure is not shaded, then 77% is not shaded.

Given any fraction with a denominator of 100, the numerator is going to be the percent.

Write the following as a percent:

Ex. 2a $\frac{39}{100}$

Ex. 2b $\frac{0.08}{100}$

Ex. 2c $\frac{5\frac{3}{7}}{100}$

Ex. 2d $\frac{100}{100}$

Solution:

a) $\dfrac{39}{100}$ = 39%.

b) $\dfrac{0.08}{100}$ = 0.08%

c) $\dfrac{5\frac{3}{7}}{100}$ = $5\frac{3}{7}$%

d) $\dfrac{100}{100}$ = 100%

The above example leads us to a very natural way of converting percents to fractions. By working it backwards, we can see that to convert a percent to a fraction, we simply put the percent over 100 or divide the percent by 100:

Objective b: Converting a percent to a fraction.

Converting a Percent to a Fraction:

$P\% = \dfrac{P}{100}$ or $P \div 100$

Write the following as a fraction:

Ex. 3a   56%   Ex. 3c   0.015%

Ex. 3b   235%   Ex. 3d $13\frac{1}{3}$%

Solution:

a)   Write 56 over 100 and reduce: $56\% = \dfrac{56}{100} = \dfrac{14}{25}$.

b)   Write 235 over 100 and reduce: $235\% = \dfrac{235}{100} = 2\dfrac{35}{100} = 2\dfrac{7}{20}$.

c)   Write 0.015 over 100. Move the decimal point, and reduce:
$0.015\% = \dfrac{0.015}{100} = \dfrac{15}{100000} = \dfrac{3}{20000}$.

d)   Divide $13\frac{1}{3}$ by 100 and simplify: $13\frac{1}{3}\% = 13\frac{1}{3} \div 100$

$= \dfrac{40}{3} \div \dfrac{100}{1} = \dfrac{40}{3} \bullet \dfrac{1}{100} = \dfrac{2}{3} \bullet \dfrac{1}{5} = \dfrac{2}{15}$.

To convert fractions into percents, instead of dividing by 100, we will multiply by 100%

Objective c: Converting a fraction to a percent.

Converting a Fraction to a Percent:

$$F = F \bullet \frac{100\%}{1}$$

Write the following as a percent:

Ex. 4a $\dfrac{2}{5}$ Ex. 4c $6\dfrac{4}{7}$

Ex. 4b $3\dfrac{3}{8}$ Ex. 4d $\dfrac{11}{12}$

Solution:

a)   Multiply $\dfrac{2}{5}$ by $\dfrac{100\%}{1}$ and simplify: $\dfrac{2}{5} \bullet \dfrac{100\%}{1} = \dfrac{2}{1} \bullet \dfrac{20\%}{1} = 40\%$.

b)   Multiply $3\dfrac{3}{8}$ by $\dfrac{100\%}{1}$ and simplify: $3\dfrac{3}{8} \bullet \dfrac{100\%}{1} = \dfrac{27}{8} \bullet \dfrac{100\%}{1}$

        $= \dfrac{27}{2} \bullet \dfrac{25\%}{1} = \dfrac{675\%}{2} = 337\dfrac{1}{2}\%$ or 337.5%.

c)   Multiply $6\dfrac{4}{7}$ by $\dfrac{100\%}{1}$ and simplify: $6\dfrac{4}{7} \bullet \dfrac{100\%}{1} = \dfrac{46}{7} \bullet \dfrac{100\%}{1}$

        $= \dfrac{4600\%}{7} = 657\dfrac{1}{7}\%$.

d)   Multiply $\dfrac{11}{12}$ by $\dfrac{100\%}{1}$ and simplify: $\dfrac{11}{12} \bullet \dfrac{100\%}{1} = \dfrac{11}{3} \bullet \dfrac{25\%}{1}$

        $= \dfrac{275\%}{3} = 91\dfrac{2}{3}\%$.

Converting between percents and decimals works in a similar fashion to converting between fractions and percents. To convert a percent to a decimal, multiply the percent by $\dfrac{1}{100}$ or 0.01.

Objective d: Converting a percent to a decimal.

Converting a Percent to a Decimal:

     $P\% = P \bullet 0.01$
     This will move the decimal point two places to the left.

We can use money as an analogy for converting percents to decimals. Since 56¢ = \$0.56, then 56% = 0.56. The percent is our cent and the dollar is our decimal.

Write the following as decimals:

Ex. 5a	765%	Ex. 5c	5.3$\overline{2}$%
Ex. 5b	7%	Ex. 5d	463.$\overline{63}$%

Solution:
a)   765% = 765(0.01) = 7.65.
b)   7% = 7(0.01) = 0.07.
c)   5.3$\overline{2}$% = 5.3$\overline{2}$(0.01) = 0.053$\overline{2}$.
d)   463.$\overline{63}$% = 463.$\overline{63}$(0.01) = 4.63$\overline{63}$ = 4.$\overline{63}$.

Just like working with fractions, to convert a decimal into a fraction, multiply the decimal by 100%.

Objective e: Converting a decimal to a percent

Converting a Decimal to a Percent:

D = D•100%
This will move the decimal two places to the right.

Again, we can use money as an analogy for converting percents to decimals. Since $0.56 = 56¢, then 0.56 = 56%. The dollar is our decimal and the percent is our cent.

Write the following as a percent:

Ex. 6a	0.9	Ex. 6d	9.$\overline{548}$
Ex. 6b	34	Ex. 6e	0.0005
Ex. 6c	0.$\overline{3}$		

Solution:
a)   0.9 = 0.9(100%) = 90%.
b)   34 = 34(100%) = 3400%.
c)   0.$\overline{3}$ = 0.333333...(100%) = 33.3333...% = 33.$\overline{3}$%
d)   9.$\overline{548}$ = 9.54854854...(100%) = 954.854854...% = 954.$\overline{854}$%.
e)   0.0005 = 0.0005(100%) = 0.05%.

Objective f: Understanding working with percents.

When a coach says, "I want you to give me 100% out on the field today," the coach means for you to give it your all. So 100% of number is all of the number. On the other hand, 50% is half of a number and 25% is a one fourth of a number.

Find the following:

Ex. 7a   100% of 57?   Ex. 7c   100% of $\frac{7}{8}$?

Ex. 7b   100% of $-$ 6.47?

    Solution:

    a)   100% of 57 = 57.

    b)   100% of 6.47 = 6.47.

    c)   100% of $\frac{7}{8}$ = $\frac{7}{8}$.

Ex. 8a   50% of 57?   Ex. 8c   50% of $\frac{7}{8}$?

Ex. 8b   50% of 6.47?

    Solution:

    a)   50% of 57 = $\frac{1}{2}$(57) = $\frac{57}{2}$ = 28.5 .

    b)   50% of 6.47 = $\frac{1}{2}$(6.47) = $\frac{6.47}{2}$ = 3.235.

    c)   50% of $\frac{7}{8}$ = $\frac{1}{2}\left(\frac{7}{8}\right)$ = $\frac{7}{16}$.

Ex. 9a   25% of 57?   Ex. 9c   25% of $\frac{7}{8}$?

Ex. 9b   25% of $-$ 6.47?

    Solution:

    a)   25% of 57 = $\frac{1}{4}$(57) = $\frac{57}{4}$ = 14.25 .

    b)   25% of 6.47 = $\frac{1}{4}$(6.47) = $\frac{6.47}{4}$ = 1.6175.

    c)   25% of $\frac{7}{8}$ = $\frac{1}{4}\left(\frac{7}{8}\right)$ = $\frac{7}{32}$.