Sect 7.1 – Converting U.S. Customary Units of Length

Objective c: Converting U.S. Units of Length by Using Unit Rates.

In this section, we will be working with the U.S. system of measurement and converting between various units. To convert from one unit to another unit, we will use unit conversion factors. To form a unit conversion factor, we start with a conversion fact (i.e., 1 ft = 12 in) and divide both sides by the value on one side of the conversion fact. If we use 1 ft = 12 in, we can divide both sides by 12 in to get:

$$\frac{1\text{ ft}}{12\text{ in}} = \frac{12\text{ in}}{12\text{ in}}$$

$$\frac{1\text{ft}}{12\text{in}} = 1$$

Notice that $\frac{1\text{ft}}{12\text{in}}$ is the same as one, so if we needed to convert 48 in into ft, we can multiply 48 in by this unit conversion factor. It does not change the value of 48 in since we are multiply by a form of one:

$$48\text{ in} = 48\text{ in} \bullet 1 = \frac{48\text{in}}{1} \bullet \frac{1\text{ft}}{12\text{in}} = \frac{48\cancel{\text{in}}}{1} \bullet \frac{1\text{ft}}{12\cancel{\text{in}}} = \frac{48}{12}\text{ ft} = 4\text{ ft.}$$

Notice that the inches divide out. We always set-up our unit conversion factors so that the units we are converting from divide out. So, if we had to convert from ft to in, we would use $\frac{12\text{in}}{1\text{ft}}$ so that the ft would divide out, leaving our answer in inches. Here are some common of length conversions within the U.S. system of measurement:

12 in = 1 ft 36 in = 3 ft = 1 yd 5280 ft = 1760 yd = 1 mi

Note, ' denotes feet and " denotes inches, thus 4' = 4 ft and 7" = 7 in.

Convert each unit as indicated and simplify:

Ex. 1 Convert 10.5 yd to ____ ft

Solution:

Since 1 yd = 3 ft, we want to write our unit conversion factor with 1 yd on the bottom:

$$10.5\text{ yd} = \frac{10.5\text{yd}}{1} \bullet \frac{3\text{ ft}}{1\text{yd}} = \frac{10.5\cancel{\text{yd}}}{1} \bullet \frac{3\text{ ft}}{1\cancel{\text{yd}}} = \frac{31.5}{1}\text{ ft} = 31.5\text{ ft}$$

Ex. 2 Convert 440 yd to ____ mi

Solution:

Since 1760 yd = 1 mi, we want to write our unit conversion factor with

1760 yd on the bottom:

440 yd = $\frac{440\,yd}{1} \bullet \frac{1mi}{1760\,yd} = \frac{440\,\cancel{yd}}{1} \bullet \frac{1mi}{1760\,\cancel{yd}} = \frac{440}{1760}$ mi = 0.25 mi

Ex. 3 Convert 177408 in to ____ mi

Solution:

First use 12 in = 1 ft to convert the inches to feet:

177408 in = $\frac{177408\,in}{1} \bullet \frac{1ft}{12\,in} = \frac{177408\,ft}{12}$ = 14,784 ft

Now, use 5280 ft = 1 mi to convert 14,784 ft into mi:

14,784 ft = $\frac{14784\,ft}{1} \bullet \frac{1mi}{5280\,ft} = \frac{14784}{5280}$ mi = 2.8 mi.

Objective d: Adding and Subtracting Mixed Units.

Many times, we see measurements expressed with mixed units. When a baby is born, the weight may be expressed as 8 pounds and 11 ounces or more simply as 8 lb 11 oz instead of 139 oz. Converting between units and mixed units is similar to converting between improper fractions and mixed numbers. Let us try some examples:

Convert each unit as indicated and simplify:

Ex. 4 1205 in to ____ yd ____ ft ____ in.

Solution:

Since 1 ft = 12 in, we will multiply $\frac{1205\,in}{1}$ by $\frac{1ft}{12\,in}$, but instead of simplifying, we will treat this as a long division problem:

$$\frac{1205\,in}{1} \bullet \frac{1ft}{12\,in} = \frac{1205}{12}\ ft$$

$$\begin{array}{r} 100 \\ 12 \overline{)\ 1205} \\ -\ 1200 \\ \hline 5 \end{array}$$

So, our answer is 100 R 5.

The 100 corresponds to the number of feet and the remainder is how many inches are leftover.

Thus, 1205 in = __100__ ft __5__ in.

Now, convert 100 ft to ____ yd ____ ft.

Since 3 ft = 1 yd, we will multiply $\frac{100\,ft}{1}$ by $\frac{1\,yd}{3\,ft}$, but instead of simplifying, we will treat this as a long division problem:

$$\frac{100\,ft}{1} \bullet \frac{1\,yd}{3\,ft} = \frac{100}{3}\ yd$$

$$\begin{array}{r} 33 \\ 3\overline{)\,100} \\ -\ 99 \\ \hline 1 \end{array}$$

So, our answer is 33 R 1.
The 33 corresponds to the number of yards and the remainder is how many feet are leftover.

Thus, 100 ft = <u>33</u> yd <u>1</u> ft and hence,
1205 in = <u>100</u> ft <u>5</u> in = <u>33</u> yd <u>1</u> ft <u>5</u> in.

Ex. 5

$$\begin{array}{rr} 3 \text{ mi} & 1100 \text{ yd} \\ +\ 5 \text{ mi} & 700 \text{ yd} \\ \hline \end{array}$$

<u>Solution:</u>
First add and then take care of the carries:

$$\begin{array}{rr} 3 \text{ mi} & 1100 \text{ yd} \\ +\ 5 \text{ mi} & 700 \text{ yd} \\ \hline 8 \text{ mi} & 1800 \text{ yd} \end{array}$$

$= 8$ mi $+ 1$ mi 40 yd
$= 9$ mi 40 yd

But, 1800 yd = 1 mi 40 yd, so add one to the 8 mi:

Ex. 6

$$\begin{array}{rrr} 8 \text{ yd} & 1 \text{ ft} & 5 \text{ in} \\ -\ 4 \text{ yd} & 2 \text{ ft} & 9 \text{ in} \\ \hline \end{array}$$

<u>Solution:</u>
Borrow one from the 1 ft, convert it to 12 in and add to the 5 in:

$$\begin{array}{rrr} & 0 \text{ ft} + \textit{12 in} & \\ 8 \text{ yd} & \cancel{1 \text{ ft}} & 5 \text{ in} \\ -\ 4 \text{ yd} & 2 \text{ ft} & 9 \text{ in} \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{rrr} 8 \text{ yd} & 0 \text{ ft} & 17 \text{ in} \\ -\ 4 \text{ yd} & 2 \text{ ft} & 9 \text{ in} \\ \hline & & 8 \text{ in} \end{array}$$

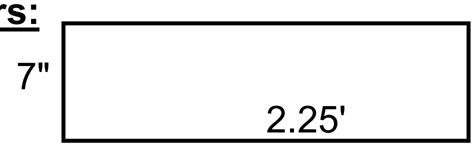
Now, borrow one from the 8 yd, convert it to 3 ft and add it to the 0 ft. Then finish the problem:

$$\begin{array}{rrr} 7 \text{ yd} + \textit{3 ft} & & \\ \cancel{8 \text{ yd}} & 0 \text{ ft} & 17 \text{ in} \\ -\ 4 \text{ yd} & 2 \text{ ft} & 9 \text{ in} \\ \hline & & 8 \text{ in} \end{array} \quad \rightarrow \quad \begin{array}{rrr} 7 \text{ yd} & 3 \text{ ft} & 17 \text{ in} \\ -\ 4 \text{ yd} & 2 \text{ ft} & 9 \text{ in} \\ \hline 3 \text{ yd} & 1 \text{ ft} & 8 \text{ in} \end{array}$$

Objective e: Applications

<u>Solve the following. Simplify all answers:</u>

Ex. 7 Find the perimeter (in inches) of the object to the right:



Solution:
First, we need to convert the feet into inches.
Since 12 in = 1 ft, we want to write our unit conversion factor with 1 ft on the bottom:

$$2.25 \text{ ft} = \frac{2.25\,\text{ft}}{1} \bullet \frac{12\,\text{in}}{1\,\text{ft}} = \frac{27}{1} \text{ in} = 27 \text{ in}$$

So, P = 7" + 2.25' + 7" + 2.25' = 7" + 27" + 7" + 27" = 68"
Thus, the perimeter is 68 inches.

Ex. 8 A road construction company can pave 43 yd 2 ft of a road per day. How much can the company pave in 5 days?
Solution:
Recall that multiplication is a shortcut for repeated addition, so we need to multiply 43 yd 2 ft by 5:

5(43 yd 2 ft) = 5 (43 yd + 2 ft) (use the distributive property)
= 5•43 yd + 5•2 ft
= 215 yd + 10 ft

Now, convert 10 ft to __ yd __ ft:

$$\frac{10\,\text{ft}}{1} \bullet \frac{1\,\text{yd}}{3\,\text{ft}} = \frac{10}{3} \text{ yd}$$

$$\begin{array}{r} 3 \\ 3\overline{)\,10} \\ \underline{-\,9} \\ 1 \end{array}$$

So, 10 ft = 3 yd 1 ft
Thus, 215 yd + 10 ft = 215 yd + 3 yd 1 ft = 218 yd 1 ft
The company can pave 218 yd 1 ft of a road in 5 days.