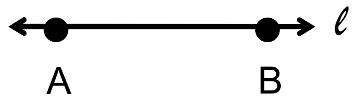
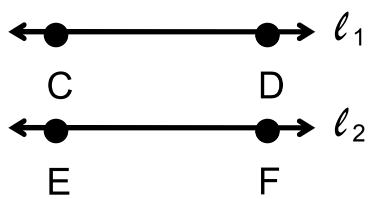
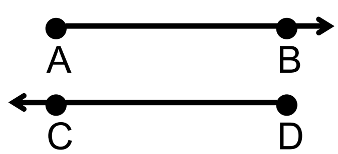
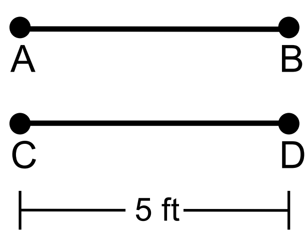
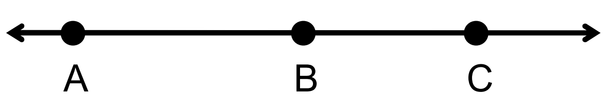
Sect 8.1 – Lines and Angles

Objective a: Basic Definitions.

Definition	**Illustration**	**Notation**
A point is a location in space. It is indicated by making a dot. Points are typically labeled with capital letters next to the dot.		Point A and point B
A line is determined by two different points and extends infinitely in **two** directions. A line can be labeled by small case letter or by two different points. If a picture contains two or more lines, subscripts can be used to denote the lines.		Line AB or line ℓ or $\overleftrightarrow{AB}$ $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ or lines ℓ_1 and ℓ_2
A ray is determined by two different points and extends infinitely in **one** direction.		Ray AB or $\overrightarrow{AB}$ Ray DC or $\overrightarrow{DC}$
A line segment is determined by two different points and extends in **no** direction. The two different points are called "endpoints." The distance between the endpoints is called the length of the line segment.		Line segment AB or $\overline{AB}$. The length of $\overline{AB}$ or AB = 5 ft. Note: $\overline{AB}$ refers to the line segment and AB refers to the length.

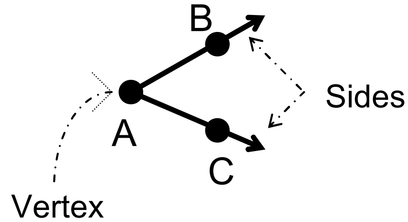
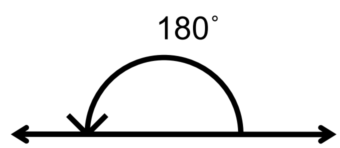
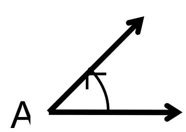
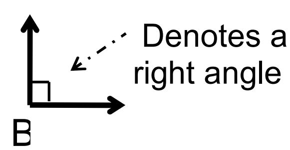
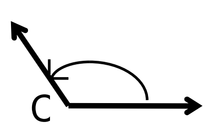
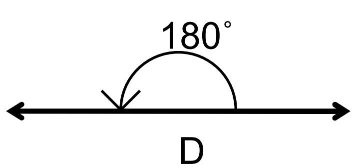
Solve the following:

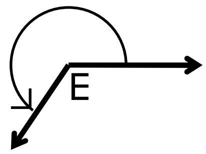
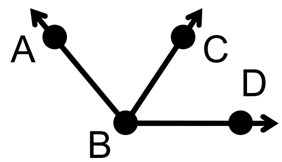
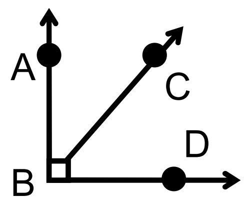
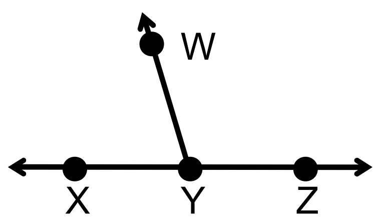
Ex 1 Given AC = 16 inches and AB = 9 inches in the diagram below, find BC.



Solution:
Since BC = AC – AB, then BC = 16 – 9 = 7 inches.

Objective b & c: Understanding types of angles.

Definition	**Illustration**	**Notation**
Two rays that share a common endpoint form an angle. Rays AC and AB form angle CAB. The common endpoint, A, is called the vertex of the angle and the two rays are called the sides of the angle. An angle is measured using a tool called a protractor. A protractor is marked in equal increments called degrees. There are 180˚ in an angle whose sides form a straight line.	 	Angle A or ∠ A Sometimes its useful to use all three points to denote an angle, i.e., “angle BAC” or “∠ CAB.” The vertex is always the middle letter in this notation. ∠ B is a straight angle. The measure of ∠ B, denoted m∠ B, is 180˚. We can say: m∠ B = 180˚
An acute angle is an angle whose measure is between 0˚ and 90˚.		∠ A is an acute angle since m∠ A < 90˚.
A right angle is an angle whose measure is exactly 90˚.		∠ B is a right angle since m∠ B = 90˚.
An obtuse angle is an angle whose measure is between 90˚ and 180˚.		∠ C is an obtuse angle since m∠ C > 90˚ and m∠ C < 180˚.
A straight angle is an angle whose measure is exactly 180˚.		∠ D is a straight angle since m∠ D = 180˚.

Definition	**Illustration**	**Notation**
A <u>reflex angle</u> is an angle whose measure is between 180˚ and 360˚.		$\angle$ E is a reflex angle since m$\angle$ E > 180˚ and m$\angle$ E < 360˚.
Two angles are called <u>adjacent angles</u> if they are side by side and have the same vertex.		$\angle$ ABC and $\angle$ CBD are adjacent angles since they are side by side and have the point B as their vertex.
Two acute angles are called <u>Complementary angles</u> if the sum of their measures is 90˚. Each is called the "complement" of the other.		$\angle$ ABC and $\angle$ CBD are complementary angles since m$\angle$ ABC + m$\angle$ CBD = 90˚.
Two angles measuring less than 180˚ are called <u>Supplementary angles</u> if the sum of their measures is 180˚. Each is called the "supplement" of the other.		$\angle$ XYW and $\angle$ WYZ are supplementary angles since m$\angle$ XYW + m$\angle$ WYZ = 180˚

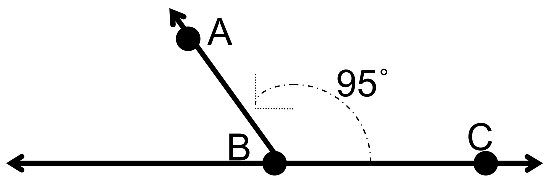
Note that complementary and supplementary angles are not always adjacent angles.

<u>Solve the following:</u>

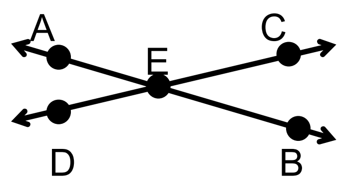
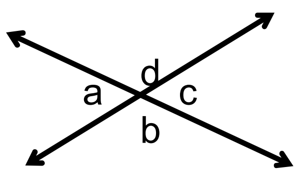
Ex. 2 Given that m $\angle$ A = 36˚, find the measure of the complement and the supplement of $\angle$ A.

<u>Solution:</u>
Since complementary angles total 90˚, then the measure of the complement of $\angle$ A is 90˚ – m$\angle$ A = 90˚ – 36˚ = 54˚.
Since supplementary angles total 180˚, then the measure of the supplement of $\angle$ A is 180˚ – m$\angle$ A = 180˚ – 36˚ = 144˚.

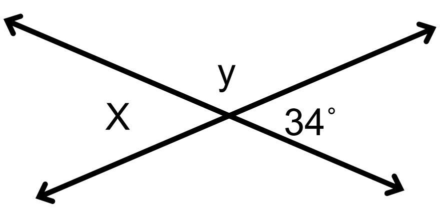
Ex. 3 Given the diagram below, find the measure of the complement and the supplement of $\angle$ ABC



Solution:
Since complementary angles have to be acute angles (less than 90˚) and the m$\angle$ ABC > 90˚, it is not possible for $\angle$ ABC to have a complement. Hence, $\angle$ ABC has no complement.
Since supplementary angles add up to 180˚, then the measure of the supplement of $\angle$ B is 180˚ – m$\angle$ B = 180˚ – 95˚ = 85˚.

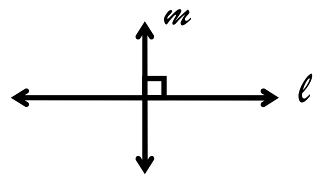
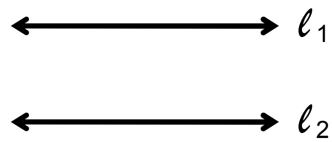
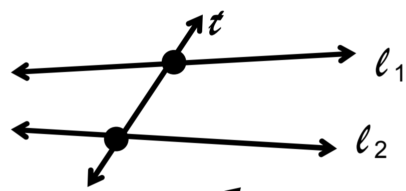
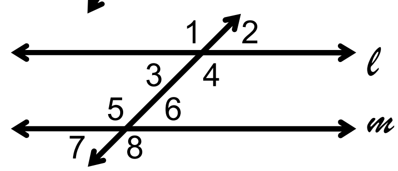
Definition	**Illustration**	**Notation**
Two or more lines are called intersecting if they "cross," "meet," or share a common point.		The two lines, $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$, intersect at point E.
Two intersecting lines form vertical angles. These angles come in pairs which have equal measures.		The vertical pairs are: $\angle$ a & $\angle$ c; $\angle$ b & $\angle$ d. m$\angle$ a = m$\angle$ c m$\angle$ b = m$\angle$ d

Ex. 4 Find the value of x and y in the following diagram:

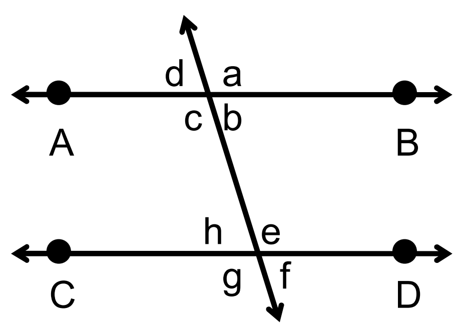


Solution:
Since $\angle$x and 34˚ are vertical angles, they are equal.
Thus, m$\angle$x = 34˚
$\angle$y is a supplementary angle to $\angle$x, so
m$\angle$y = 180˚ – m$\angle$x = 180˚ – 34˚ = 146˚

Objective d: Parallel and Perpendicular Lines.

Definition	**Illustration**	**Notation**
Two intersecting lines (or rays or line segments) that form right angles (90˚) are called perpendicular lines.		Line ℓ is perpendicular to line m or $\ell \perp m$.
Two or more lines are called parallel lines if they never "cross", "meet", or have no points in common.		Lines ℓ_1 and ℓ_2 are parallel or $\ell_1 \parallel \ell_2$.
A transversal is a line that intersects two or more different lines at different points. If the transversal intersects two parallel lines, it produces a total eight angles with some special properties.	 	Line t is the transversal since it intersects ℓ_1 & ℓ_2 at two different points. If $\ell \parallel m$, then $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8$ $m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7$

Ex. 5 Given that $m\angle b = 63^\circ$ in the diagram below, find all the other angles. Assume that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

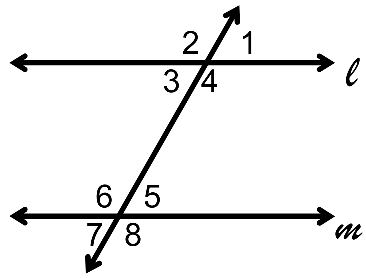


Solution:

Since $m\angle f = m\angle h = m\angle d = m\angle b$, then $m\angle f = m\angle h = m\angle d = 63^\circ$. Since $\angle c$ and $\angle b$ are supplementary angles,
$m\angle c = 180^\circ - m\angle b = 180^\circ - 63^\circ = 117^\circ$.
Hence, $m\angle g = m\angle e = m\angle a = m\angle c = 117^\circ$.

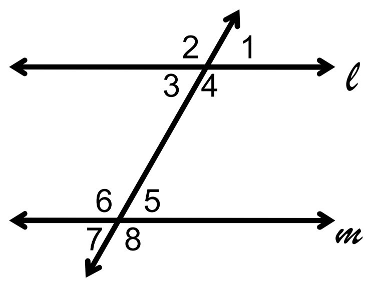
Definition	**Illustration**	**Notation**

Corresponding angles are those angles that share the **same location** in their respective intersections. If two lines that are cut by a transversal are parallel, then the corresponding angles are equal.



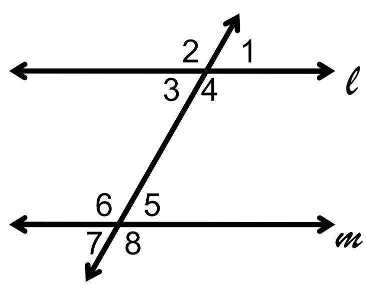
There are four pairs of corresponding angles. They are:
$\angle 1$ & $\angle 5$; $\angle 2$ & $\angle 6$;
$\angle 3$ & $\angle 7$; $\angle 4$ & $\angle 8$.
If $\ell \parallel m$, then
$m\angle 1 = m\angle 5$;
$m\angle 2 = m\angle 6$;
$m\angle 3 = m\angle 7$;
$m\angle 4 = m\angle 8$.

Alternate interior angles are those angles inside the two lines sharing the opposite locations, i.e., top - left and bottom - right. If the two lines are parallel, then the alternate interior angles are equal.



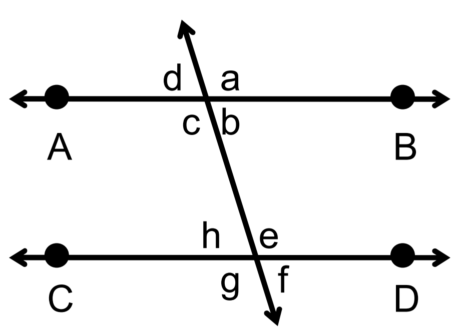
There are two pairs of alternate interior angles. They are: $\angle 3$ & $\angle 5$;
$\angle 4$ & $\angle 6$.
If $\ell \parallel m$, then
$m\angle 3 = m\angle 5$;
$m\angle 4 = m\angle 6$.

Alternate exterior angles are those angles outside the two lines sharing the opposite locations, i.e., top - left and bottom - right. If the two lines are parallel, then the alternate exterior angles are equal.



There are two pairs of alternate exterior angles. They are: $\angle 1$ & $\angle 7$;
$\angle 2$ & $\angle 8$.
If $\ell \parallel m$, then
$m\angle 1 = m\angle 7$;
$m\angle 2 = m\angle 8$.

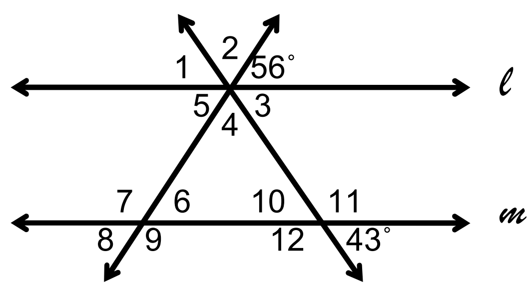
Ex. 6 Using the diagram below, fill in the following blanks. Assume that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.



a) $\angle a$ and $\angle e$ are ________________ angles.
b) $\angle d$ and $\angle f$ are ________________ angles.
c) $\angle g$ and $\angle e$ are ________________ angles.
d) $\angle h$ and $\angle b$ are ________________ angles.
e) $\angle d$ and $\angle c$ are ________________ angles.

Solution:
a) $\angle a$ and $\angle e$ are corresponding angles.
b) $\angle d$ and $\angle f$ are alternate exterior angles.
c) $\angle g$ and $\angle e$ are vertical angles.
d) $\angle h$ and $\angle b$ are alternate interior angles.
e) $\angle d$ and $\angle c$ are supplementary angles.

Ex. 7 In the diagram below, find all the missing angles. Assume $\ell \parallel m$.



Solution:
Since $\angle 5$ and 56˚ are vertical angles, $m\angle 5 = 56^\circ$. Since $\angle 1$ and 43˚ are alternate exterior angles, $m\angle 1 = 43^\circ$. But $\angle 1$ and $\angle 3$ are vertical angles, thus $m\angle 3 = 43^\circ$. $\angle 1$, $\angle 2$, and 56˚ form a straight line and $m\angle 1 = 43^\circ$, hence $180^\circ = 56^\circ + m\angle 2 + 43^\circ$. Solving for $m\angle 2$ yields $m\angle 2 = 81^\circ$. Yet, $\angle 2$ and $\angle 4$ are vertical angles which means $m\angle 4 = 81^\circ$. Since $\angle 5$ corresponds to $\angle 8$, $m\angle 8 = m\angle 5 = 56^\circ$. But $\angle 6$ and $\angle 8$ are vertical angles, so $m\angle 6 = 56^\circ$. $\angle 7$ and $\angle 8$ are supplementary angles, therefore $m\angle 7 = 180^\circ - m\angle 8 = 180^\circ - 56^\circ = 124^\circ$. $\angle 7$ and $\angle 9$ are vertical angles and thus $m\angle 9 = 124^\circ$. $\angle 10$ and 43˚ are vertical angles which means $m\angle 10 = 43^\circ$. Since $\angle 10$ and $\angle 11$ are supplementary angles, $m\angle 11 = 180^\circ - 43^\circ = 137^\circ$. But, $\angle 11$ and $\angle 12$ are vertical angles, so $m\angle 12 = 137^\circ$. Therefore:

$m\angle 1 = 43^\circ$	$m\angle 2 = 81^\circ$	$m\angle 3 = 43^\circ$	$m\angle 4 = 81^\circ$
$m\angle 5 = 56^\circ$	$m\angle 6 = 56^\circ$	$m\angle 7 = 124^\circ$	$m\angle 8 = 56^\circ$
$m\angle 9 = 124^\circ$	$m\angle 10 = 43^\circ$	$m\angle 11 = 137^\circ$	$m\angle 12 = 137^\circ$