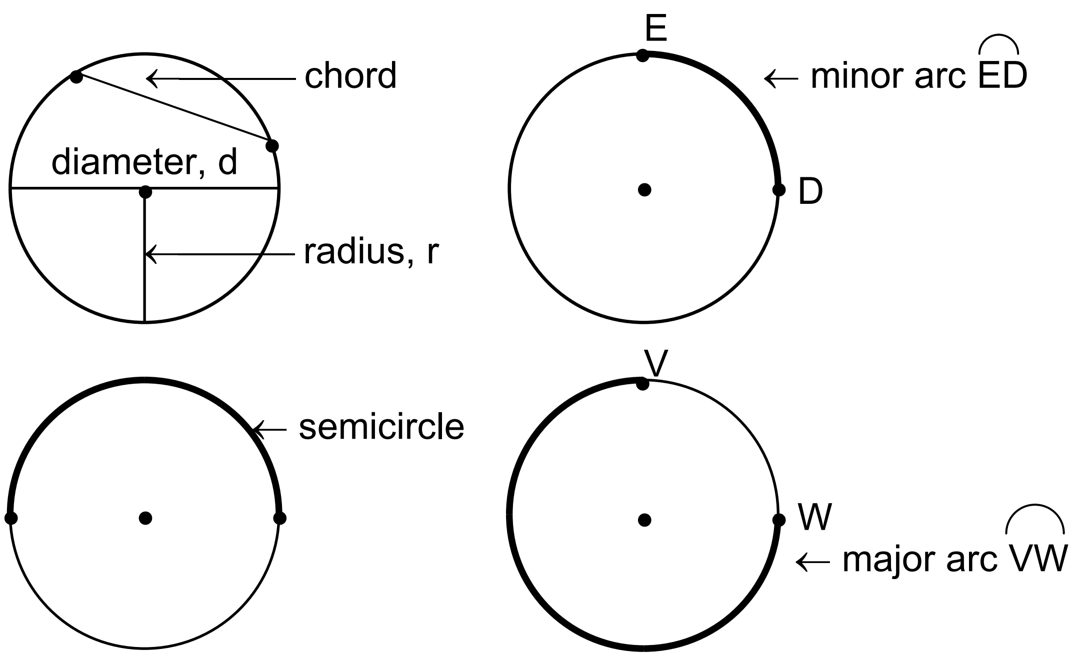
Sect 8.4- Circles, Circumference, and Area

Objective a: Basic definitions.

A **circle** is a set of all points in a plane that are equal distance from a center point. The distance from that center point to a point on the circle is called the **radius** of the circle. A **chord** is any line segment whose endpoints are two different points on the circle. If the chord passes through the center of a circle, then it is called the **diameter** of a circle. Recall that the diameter is twice the radius (d = 2r). Any part of a circle is called an **arc**. If the endpoints of an arc also happen to be the endpoints of the diameter of the circle, then it is called a **semicircle**. A **minor arc** is smaller than a semicircle while a **major arc** is larger than a semicircle.

The arc AB is denoted $\overset{\frown}{AB}$.



Objective b & c: The Circumference and Area of a Circle.

When we discuss the "perimeter" of a circle, we call it the circumference of the circle. We find the circumference and area of a circle by using the following formulas:

Circumference of a Circle

If d is the diameter of a circle and r is the radius of the circle, then the circumference, C, of the circle is

$$C = \pi d \quad \text{or} \quad C = 2\pi r \qquad \text{where } \pi \approx 3.1415926535\ldots$$

Area of a Circle

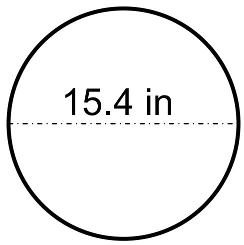
If r is the radius of a circle, then the area, A, of the circle is

$A = \pi r^2$

Find the circumference and the area of the following:

(For calculations involving π, give both the exact answer and the approximate answer).

Ex. 1



Solution:

We will use $C = \pi d$ to find the circumference since we are given the diameter:

$C = \pi d = \pi(15.4) = 15.4\pi$ in.

The 15.4π in is are exact answer. To approximate the answer, replace π by ≈ 3.14:

$15.4\pi \approx 15.4(3.14) = 48.356$ in.

To find the area, first divide 15.4 by 2 to get the radius.

$r = 15.4 \div 2 = 7.7$ in.

Now use $A = \pi r^2$:

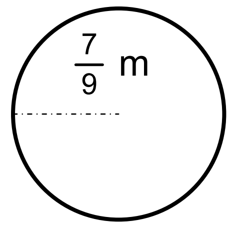
$A = \pi(7.7)^2 = 59.29\pi \text{ in}^2$. So, $59.29\pi \text{ in}^2$ is the exact answer.

To approximate the answer, replace π by 3.14:

$59.29\pi \approx 59.29(3.14)$

$= 186.1706 \text{ in}^2$

Ex. 2



Solution:

We will use $C = 2\pi r$ to find the circumference since we are given the radius:

$C = 2\pi(\frac{7}{9}) = \frac{2}{1}\pi(\frac{7}{9}) = \frac{14}{9}\pi$ m.

The $\frac{14}{9}\pi$ m is the exact answer

To approximate the answer, replace π by $\approx \frac{22}{7}$:

$\frac{14}{9}\pi \approx \frac{14}{9}(\frac{22}{7}) = \frac{2}{9}(\frac{22}{1}) = \frac{44}{9}$ m.

To find the area, replace r by $\frac{7}{9}$:

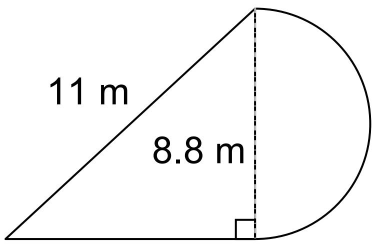
$A = \pi r^2 = \pi\left(\frac{7}{9}\right)^2 = \frac{49}{81}\pi \text{ m}^2$.

So, $\frac{49}{81}\pi \text{ m}^2$ is the exact answer.

To find the approximate answer, replace π by $\frac{22}{7}$:

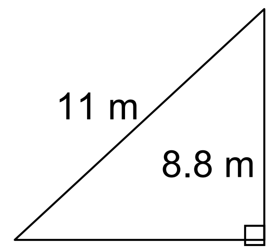
$\frac{49}{81}\pi \approx \frac{49}{81}\bullet\frac{22}{7} = \frac{7}{81}\bullet\frac{22}{1} = \frac{154}{81} \text{m}^2$.

Ex. 3



<u>Solution:</u>

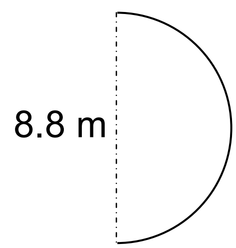
To find the perimeter, we will first need to find the base of the right triangle. Using the Pythagorean Theorem:



$$(11)^2 = (8.8)^2 + b^2$$
$$121 = 77.44 + b^2$$
$$\underline{-\,77.44 = -\,77.44\quad}$$
$$43.56 = b^2$$

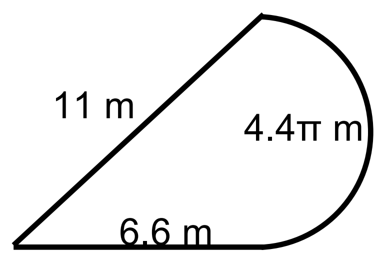
$b = \pm\sqrt{43.56} = \pm\, 6.6$ m.

So, the base is 6.6 m.



Next, we will need to find the length of the semicircle. For a full circle, the circumference is $C = \pi d$. Since we have a semicircle, we will divide our answer by 2:

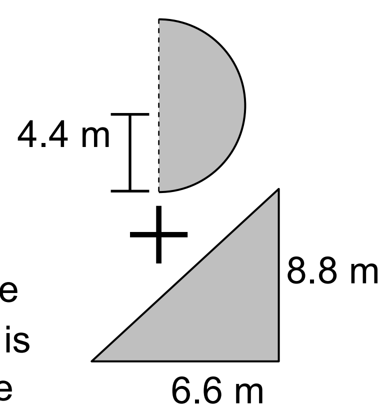
$C = \pi d = \pi(8.8) = 8.8\pi$ m. Dividing by two, we get: $8.8\pi \div 2 = 4.4\pi$ m.



Now, we add up the sides :

$P = 6.6 + 11 + 4.4\pi = (17.6 + 4.4\pi)$ m.

The exact answer is $(17.6 + 4.4\pi)$ m. Notice we did not use the 8.8 m since it is not along the outside of the figure. The approximate answer is $17.6 + 4.4\pi \approx 17.6 + 4.4(3.14)$ $= 17.6 + 13.816 = 31.416$ m.

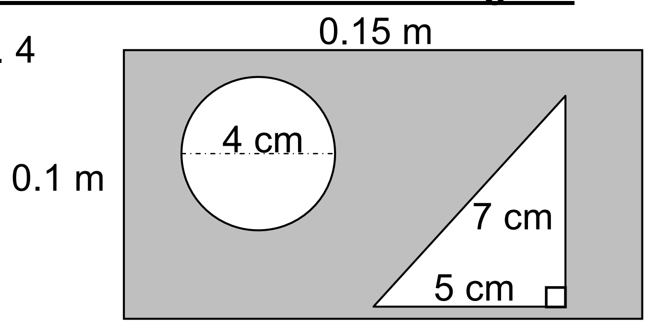


For the area, we will add the area of the triangle to the area of the semicircle. Since the diameter of the circle is 8.8 m, its radius is $8.8 \div 2 = 4.4$ m. The area for a circle is $A = \pi r^2$, so we will divide the answer by 2: $\pi r^2 = \pi(4.4)^2 = 19.36\pi\ m^2$. Dividing by 2 , we get: $19.36\pi \div 2 = 9.68\pi\ m^2$. Since the base is 6.6 m and the height is 8.8 m, the area of the

triangle is $\frac{1}{2}(6.6)(8.8) = 29.04 \text{ m}^2$. Thus, the total area is $(9.68\pi + 29.04) \text{ m}^2$. The approximate answer is:
$9.68\pi + 29.04 \approx 9.68(3.14) + 29.04 = 59.4352 \text{ m}^2$.

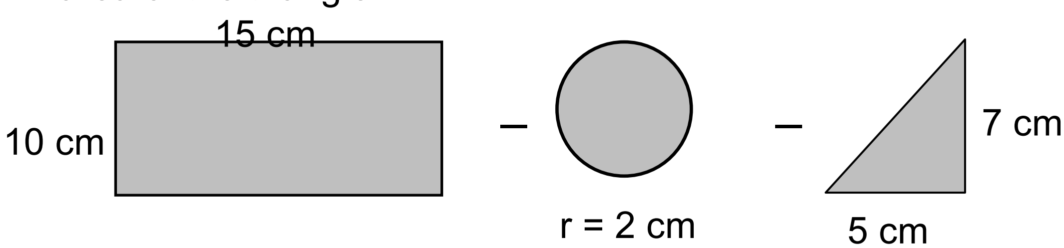
<u>Find the area of the shaded region:</u>

Ex. 4



<u>Solution:</u>
First convert the meters into centimeters:
0.1 m = 0.10 = 10 cm and 0.15 m = 0.15 = 15 cm

The area of the shade region is equal to the area of the rectangle minus the area of the circle (the radius is 4 ÷ 2 = 2 cm) and the area of the triangle:



$$A = Lw \qquad - \pi r^2 \qquad - \frac{1}{2}bh$$
$$= (15)(10) \qquad - \pi(2)^2 \qquad - \frac{1}{2}(5)(7)$$
$$= (15)(10) - \pi(2)^2 - \frac{1}{2}(5)(7)$$
$$= 150 - 4\pi - 17.5$$
$$= (132.5 - 4\pi) \text{ cm}^2 \longleftarrow \text{Exact answer.}$$

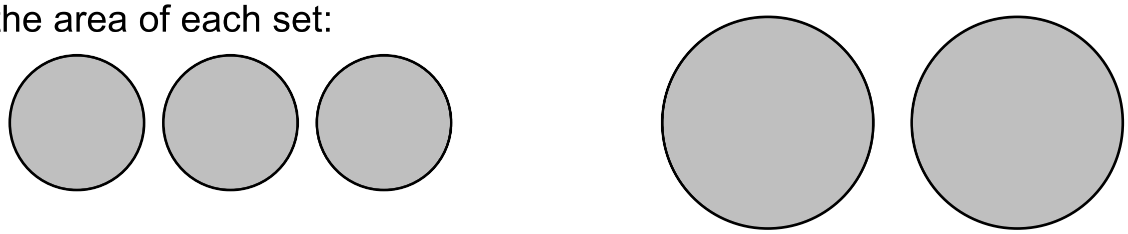
$$\approx 132.5 - 4(3.14)$$
$$= 132.5 - 12.56$$
$$= 119.94 \text{ cm}^2 \longleftarrow \text{Approximate Answer}$$

Solve the following:

Ex. 5 For the same price, Rosa can buy three twelve-inch pizzas or two sixteen-inch pizzas. a) Which set of pizzas is the better buy? b) What percent (to the nearest tenth) more pizza does she get with the better buy?

Solution:

a) For pizza, we will need to find the total area of the three 12-in pizzas and compare it to the total area of the two 16-in pizzas. The set of pizzas with the most area will be the better buy. The size of the pizzas refers to its diameter. So, the radius of the 12-in pizza is 6 in and the radius of the 16-in pizza is 8 in. Now, we can calculate the area of each set:



The area of a circle is $A = \pi r^2$.

For the medium set of pizzas, we will replace r by 6 in and calculate the area. Since we have three pizzas, we will multiply the answer by three to get the total area:

$A = \pi r^2 = \pi(6)^2 = 36\pi$ in^2.

Multiplying by 3, we get:

$3 \bullet 36\pi = 108\pi$ in^2.

For the large set of pizzas, we will replace r by 8 in and calculate the area. Since we have two pizzas, we will multiply the answer by two to get the total area:

$A = \pi r^2 = \pi(8)^2 = 64\pi$ in^2.

Multiplying by 2, we get:

$2 \bullet 64\pi = 128\pi$ in^2.

Since 128π in^2 > 108π in^2, the two large pizzas are a better buy.

b) The two large pizzas offer $128\pi - 108\pi = 20\pi$ in^2 more area than the three medium pizzas. We want to find what percent of 108π is 20π:

$20\pi = P(108\pi)$

$20\pi = (108\pi)P$ (Divide both sides by 108π)

$\frac{20\pi}{108\pi} = \frac{(108\pi)P}{108\pi}$ (The π's divide out)

$P = 0.18518 \ldots = 18.518 \ldots \%$

$\approx 18.5\%$.