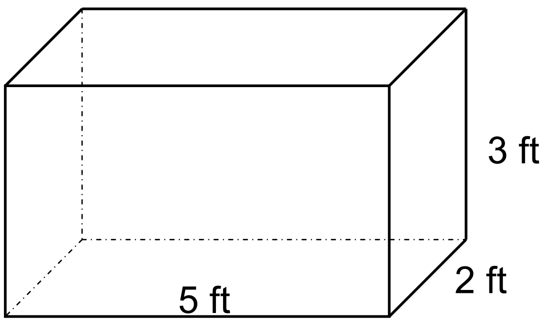
# Sect 8.5 - Volume

Objective a & b: Understanding Volume of Various Solids

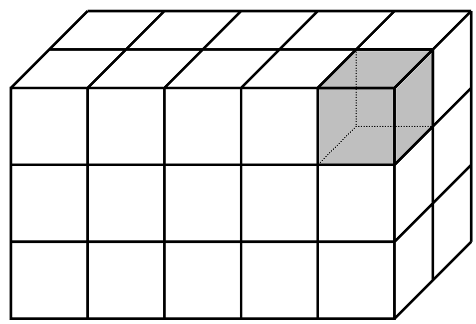
The **Volume** is the amount of space a three – dimensional object occupies. Volume is measured in cubic units such as $in^3$ or $cm^3$. We use the idea of volume to determine how much capacity a refrigerator has or how much water is flowing down a river.

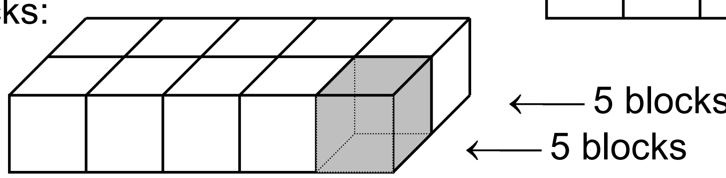
**Find the volume of the following:**

Ex. 1




Solution:
To solve this, we need to think of how many 1 ft by 1 ft by 1 ft cubes we would need to use to build a solid measuring 5 ft by 2 ft by 3 ft. To do the first layer, we would need two rows of five blocks or ten blocks:



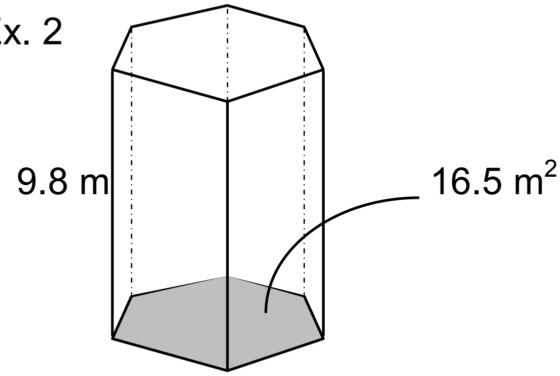



Notice that the number of blocks needed for this layer is numerically equal to the area of the base: B = Lw = 5(2) = 10 ft$^2$. We have three layers with each layer requiring 10 blocks, so we will need 3(10) or 30 block. So, the volume is equal to the area of the base times the height of the figure or Bh. This will work for any type of prism. Thus, the volume is 30 ft$^3$.

A **prism** is a solid figure whose bases or ends have the same size and shape and are parallel to one another, and whose sides are parallelograms. The formula for the volume of a prism is V = Bh, where B is the area of the base.

**<u>Find the volume of the following:</u>**

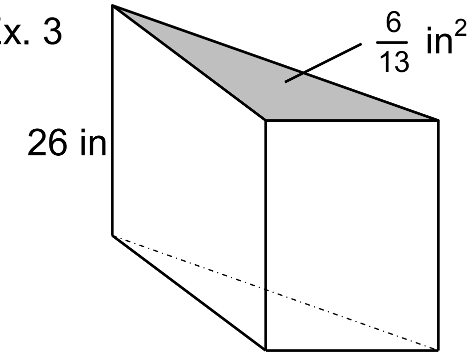
Ex. 2




<u>Solution:</u>
Since this is a prism and we are given the area of the base and the height, the volume is

$V = Bh = (16.5)(9.8) = 161.7 \text{ m}^3$.

Ex. 3

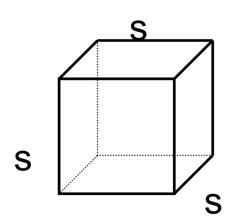



<u>Solution:</u>
Since this is a prism and we are given the area of the base and the height, the volume is $V = Bh = \left(\frac{6}{13}\right)(26)$
$= \left(\frac{6}{13}\right)\left(\frac{26}{1}\right) = \left(\frac{6}{1}\right)\left(\frac{2}{1}\right)$
$= 12 \text{ in}^3$.

For a cube, the length of the sides are all the same length. The base is a square so its area is $s^2$ and the height is equal to s, so it volume is $V = Bh = s^2 \bullet s = s^3$.

**Volume of a Cube**

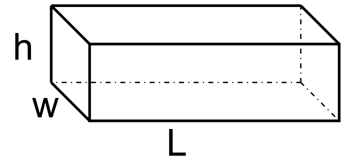
In general, if s is the length of the side of the cube, then the formula for the volume of a cube is $V = s^3$.




For a rectangular solid or prism (a box), the area of the base is equal to Lw and the height is h. Thus, the volume is V = Bh = Lwh.

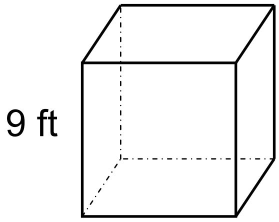
**Volume of a Rectangular Solid or Prism**

In general, if L is the length, w is the width and h is the height of the rectangular solid, then the formula for the volume is V = Lwh.

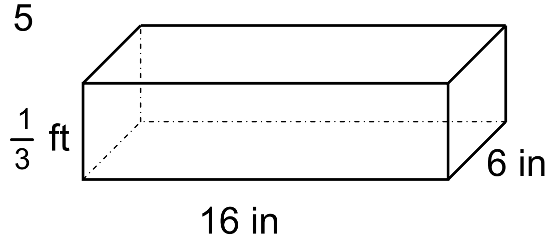


**<u>Find the volume of the following:</u>**

Ex. 4




Ex. 5




<u>Solution:</u>

The volume of a cube is $s^3$.

Thus, $V = s^3 = (9)^3 = 729\ \text{ft}^3$.

<u>Solution:</u>

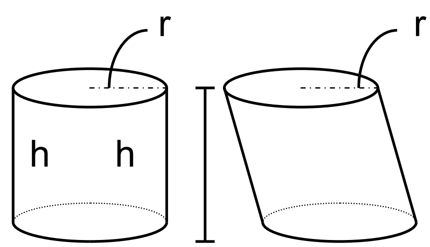
First convert $\frac{1}{3}$ ft into inches:

$\frac{1}{3} = \frac{1\text{ft}}{3} \bullet \frac{12\text{in}}{1\text{ft}} = 4$ in. The volume of a rectangular solid is $V = Lwh$ $= (16)(6)(4) = 384\ \text{in}^3$.

We can think of a cylinder as a type of prism. Its base is a circle so the area of the base is $\pi r^2$ and the height is h. Thus, the volume for a cylinder is $V = Bh = \pi r^2 h$.

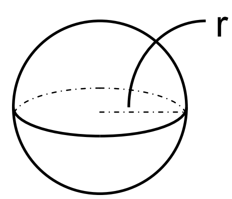
**Volume of a Cylinder**

In general, if r is the radius of the cylinder and h is the height of the cylinder, then the volume is given by $V = \pi r^2 h$.




**Volume of a Sphere**

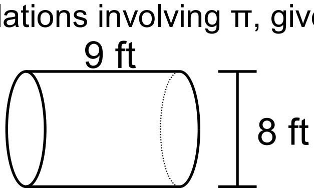
In general, if r is the radius of the sphere, then the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$



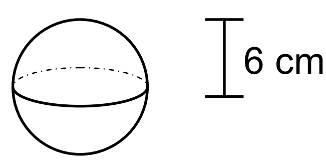

**<u>Find the volume of the following:</u>**

(For calculations involving π, give both the exact answer and the approximate answer).

Ex. 6




Ex. 7




<u>Solution:</u>

First, we divide the diameter by 2 to get the radius: $8 \div 2 = 4$ ft.

<u>Solution:</u>

Since the radius is 6 cm, then the volume is $V = \frac{4}{3}\pi r^3$

Think of this as a cylinder lying down on its side. So, the height is 9 ft. Thus, the volume is $V = \pi r^2 h$
$= \pi(4)^2(9) = \pi(16)(9) = 144\pi \text{ ft}^3$.
$\approx 144(3.14) = 452.16 \text{ ft}^3$.
Exact Answer: $144\pi \text{ ft}^3$
Approximate: $452.16 \text{ ft}^3$

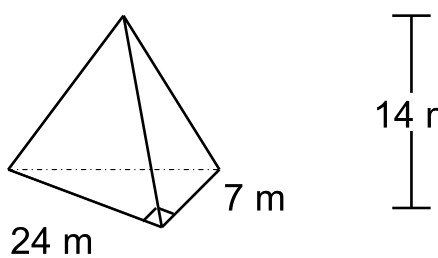
$= \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216)$
$= \frac{4}{3}\pi\left(\frac{216}{1}\right) = \frac{4}{1}\pi\left(\frac{72}{1}\right)$
$= 288\pi \text{ cm}^3 \approx 288(3.14)$
$= 904.32 \text{ cm}^3$.

Exact Answer: $288\pi \text{ cm}^3$
Approximate: $904.32 \text{ cm}^3$

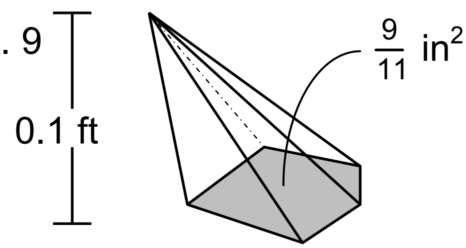
A **Pyramid** is a solid figure with a polygon as a base and sides that are triangles which meet at a common point at the top opposite the base. The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base.

**Find the volume of the following:**

Ex. 8




Ex. 9



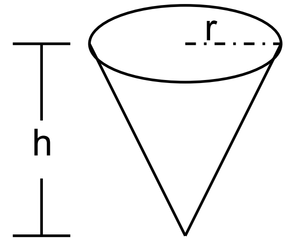

Solution:
First, we need to find the area of the base. Since the base is a triangle with b = 7 m and h = 24, then the area of the triangle is $B = \frac{1}{2}bh = \frac{1}{2}(7)(24)$
$= 84 \text{ m}^2$. The volume of the pyramid is $V = \frac{1}{3}Bh = \frac{1}{3}(84)(14)$
$= 392 \text{ m}^3$.

Solution:
First, we need to convert the 0.1 ft into inches:
$0.1 \text{ ft} = \frac{1\text{ft}}{10} \bullet \frac{12\text{in}}{1\text{ft}} = \frac{6}{5}$ in.
Since $B = \frac{9}{11}$ in$^2$ & $h = \frac{6}{5}$ in, then the volume of the pyramid is $V = \frac{1}{3}Bh$
$= \frac{1}{3}Bh = \frac{1}{3}\left(\frac{9}{11}\right)\left(\frac{6}{5}\right)$
$= \frac{18}{55}$ in$^3$.

If we think of a cone as being like a pyramid with a circular base, then the area of the base is $B = \pi r^2$. So, the volume of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$.

**Volume of a Cone**

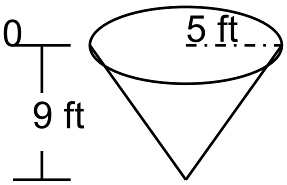
In general, is r is the radius of the cone and h is the height of the cone, then the volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$.




**<u>Find the volume of the following:</u>**

(For calculations involving π, give both the exact answer and the approximate answer).

Ex. 10

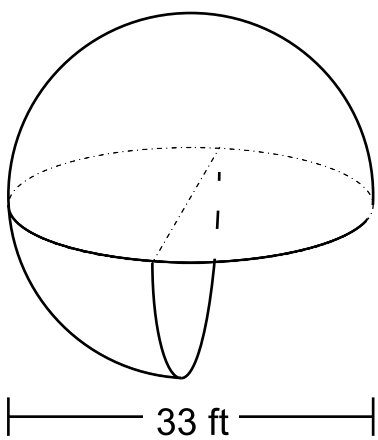



<u>Solution:</u>

Since h = 9 ft and r = 5 ft, we can plug directly into $V = \frac{1}{3}\pi r^2 h$ to get the answer: $V = \frac{1}{3}\pi(5)^2(9) = \frac{1}{3}\pi(25)(9) = 75\pi \text{ ft}^3$ (Exact answer)

$\approx 75(3.14) = 235.5 \text{ ft}^3$ (Approximate answer).

Ex. 11




<u>Solution:</u>

We have three-fourths of a sphere so the volume is three-fourths of the volume of a full sphere. Since the diameter is 33 ft, the radius is 33 ft ÷ 2 = 16.5 ft. The volume for a full sphere with radius 16.5 ft is:

$$V = \frac{4\pi r^3}{3} = \frac{4\pi(16.5)^3}{3} \text{ ft}^3 = \frac{17968.5\pi}{3} \text{ ft}^3 = 5989.5\pi \text{ ft}^3$$

So, for three-fourths of a sphere, the volume is:

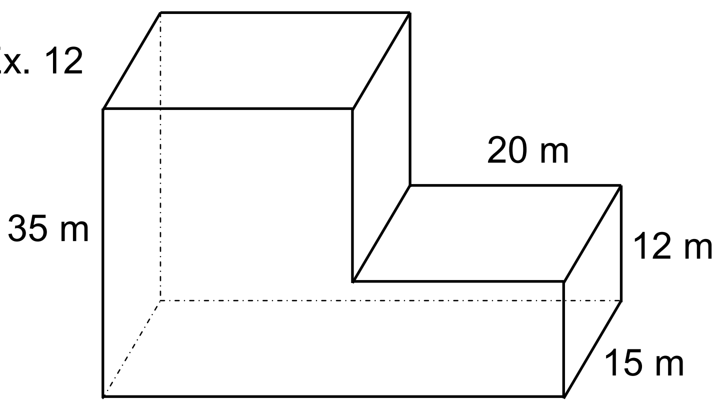
$$V = \frac{3}{4}(5989.5\pi \text{ ft}^3) = 4492.125\pi \text{ ft}^3 \approx 14{,}105.2715 \text{ ft}^3$$

Exact Answer: $4492.125\pi \text{ ft}^3$
Approximation: $\approx 14{,}105.2715 \text{ ft}^3$

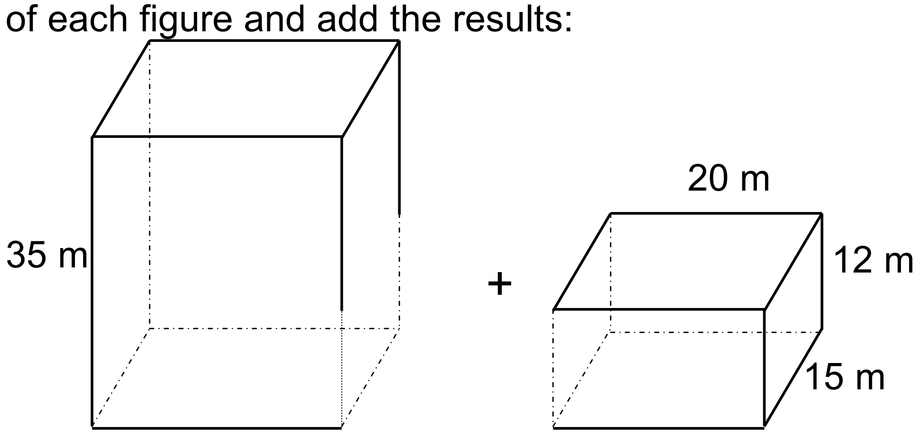
A **Composite Figure** is a geometric figure that consists of two or more basic geometric figures. In this section, we are interested in finding the volume of these figures. Keep in mind that when we are talking about the volume of any figure, we want to find the total amount of space the object occupies; i.e., how much soda a soda can holds or how much concrete needs to be poured to make a driveway. The key to solving a composite figure problem is to split it apart into a series of basic figures. Let's look at some examples.

**Find the volume of the following:**
(For calculations involving π, give both the exact answer and the approximate answer)

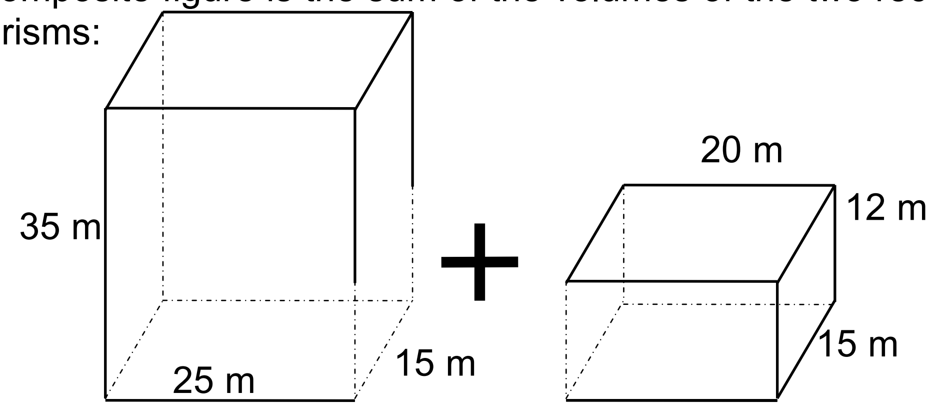
Ex. 12




Solution:
We can split this figure into two rectangular prisms, find the volume of each figure and add the results:

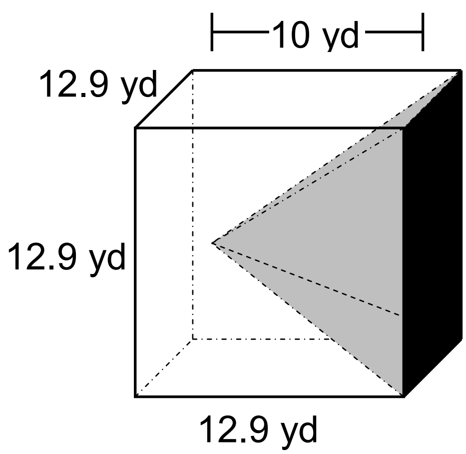


Since the total length of the figure is 45 m and the length of the smaller rectangular prism is 20 m, then the length of the larger prism is 45 m – 20 m = 25 m. Also, the width of the larger prism is equal to the width of the smaller prism which is 15 m. The volume of the composite figure is the sum of the volumes of the two rectangular prisms:



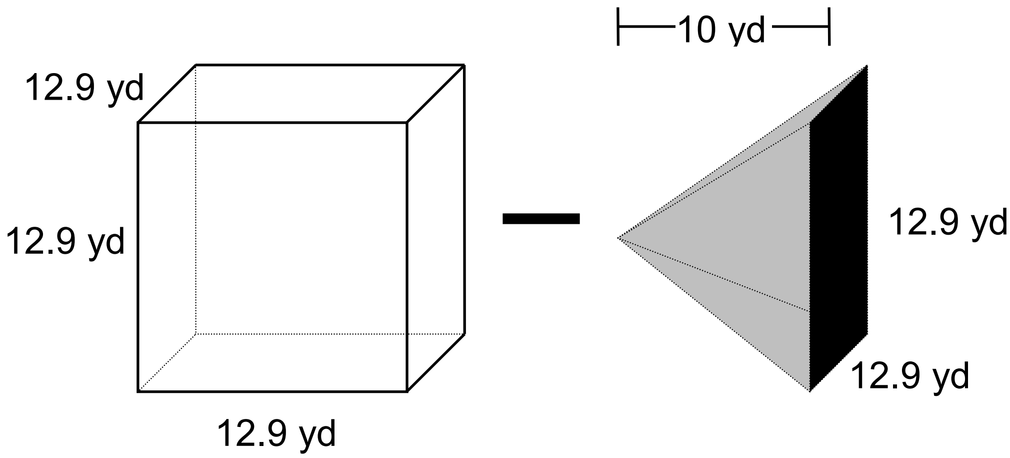

$$V = Lwh + Lwh$$
$$= [(25)(15)(35) + (20)(15)(12)]\ m^3$$
$$= [13125 + 3600]\ m^3$$
$$= 16{,}725\ m^3$$

Ex. 13




Solution:

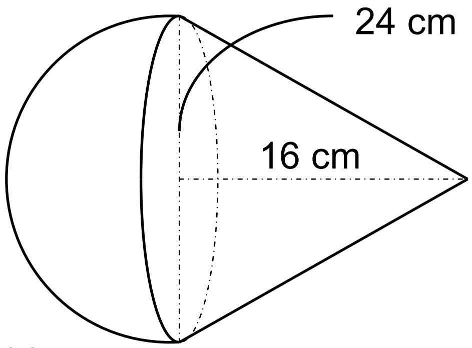
We can split this figure into a cube and a pyramid. Here, the pyramid has been "tunneled out" of the cube. So, to find the volume of the composite figure, we will subtract the volume of the pyramid from the volume of the cube:




Since the base of the pyramid is a square, its area is B = $s^2 = (12.9)^2 = 166.41$ yd$^2$. The volume of the composite figure is the difference between the volume of the cube and the volume of the pyramid:

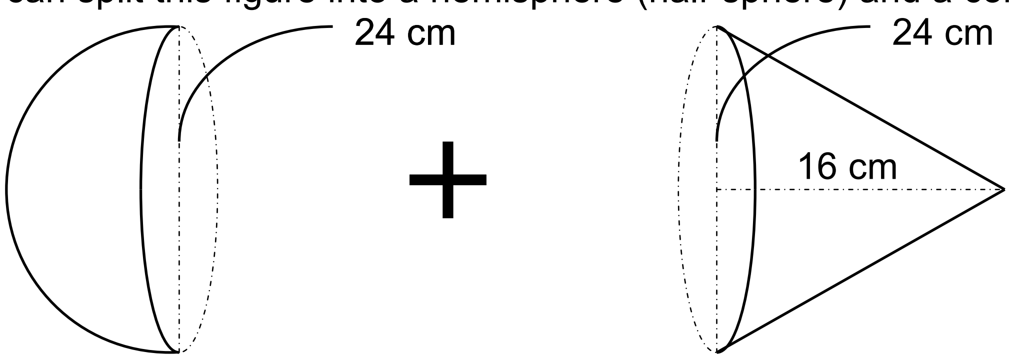
$$V = s^3 - \frac{1}{3}Bh = (12.9)^3 - \frac{1}{3}(166.41)(10)$$
$$= 2146.689 - 554.7 = 1591.989 \text{ yd}^3.$$

Ex. 14




Solution:

We can split this figure into a hemisphere (half-sphere) and a cone:




Since the diameter of the cone is equal to the diameter of the hemisphere (which is 24 cm), the radius of the both the cone and the hemisphere is 24 cm ÷ 2 = 12 cm. The volume of the composite figure is the sum of the volume of the cone and half of the volume of the sphere:

$V = \frac{1}{3}\pi r^2 h + \frac{1}{2}\bullet\left(\frac{4\pi r^3}{3}\right) = \frac{1}{3}\pi r^2 h + \frac{2\pi r^3}{3} = \frac{1}{3}\pi(12)^2(16) + \frac{2\pi(12)^3}{3}$

$= 768\pi + 1152\pi = 1920\pi\ \text{cm}^3 \approx 6028.8\ \text{cm}^3$.

Exact Answer: $1920\pi\ \text{cm}^3$

Approximation: $\approx 6028.8\ \text{cm}^3$