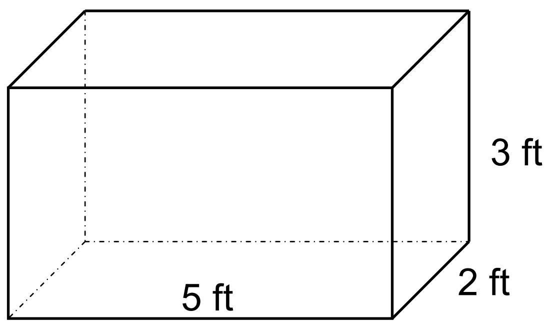
# Sect 9.7 - Volume and Surface Area

Objective a: Understanding Volume of Various Solids

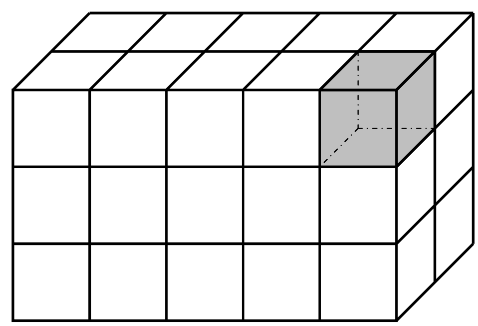
The **Volume** is the amount of space a three – dimensional object occupies. Volume is measured in cubic units such as $in^3$ or $cm^3$. We use the idea of volume to determine how much capacity a refrigerator has or how much water is flowing down a river.

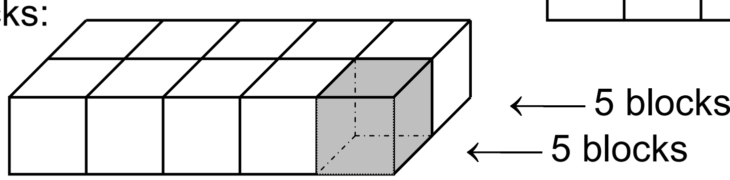
**Find the volume of the following:**

Ex. 1




Solution:
To solve this, we need to think of how many 1 ft by 1 ft by 1 ft cubes we would need to use to build a solid measuring 5 ft by 2 ft by 3 ft. To do the first layer, we would need two rows of five blocks or ten blocks:



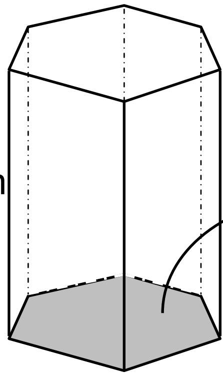



Notice that the number of blocks needed for this layer is numerically equal to the area of the base: B = Lw = 5(2) = 10 $ft^2$. We have three layers with each layer requiring 10 blocks, so we will need 3(10) or 30 block. So, the volume is equal to the area of the base times the height of the figure or Bh. This will work for any type of prism. Thus, the volume is 30 $ft^3$.

A **prism** is a solid figure whose bases or ends have the same size and shape and are parallel to one another, and whose sides are parallelograms. The formula for the volume of a prism is V = Bh, where B is the area of the base.

**Find the volume of the following:**

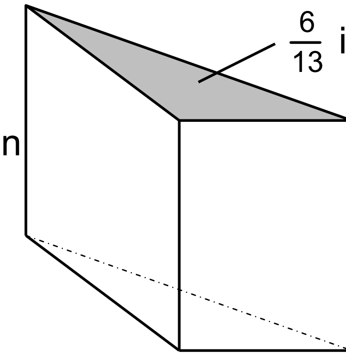
Ex. 2




Solution:
Since this is a prism and we are given the area of the base and the height, the volume is

$V = Bh = (16.5)(9.8) = 161.7\ m^3$.

Ex. 3




Solution:
Since this is a prism and we are given the area of the base and the height, the volume is $V = Bh = \left(\frac{6}{13}\right)(26)$

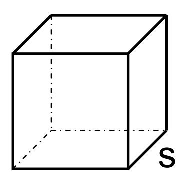
$= \left(\frac{6}{13}\right)\left(\frac{26}{1}\right) = \left(\frac{6}{1}\right)\left(\frac{2}{1}\right)$

$= 12\ in^3$.

For a cube, the lengths of the sides are all the same length. The base is a square so its area is $s^2$ and the height is equal to s, so it volume is V = Bh = $s^2 \bullet s = s^3$.

**Volume of a Cube**

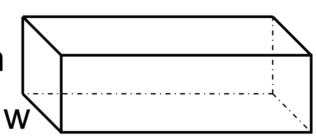
In general, if s is the length of the side of the cube, then the formula for the volume of a cube is $V = s^3$.




For a rectangular solid or prism (a box), the area of the base is equal to Lw and the height is h. Thus, the volume is V = Bh = Lwh.

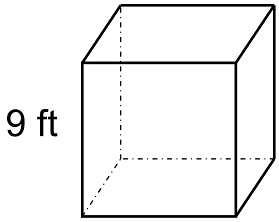
**Volume of a Rectangular Solid or Prism**

In general, if L is the length, w is the width and h is the height of the rectangular solid, then the formula for the volume is V = Lwh.



**Find the volume of the following:**

Ex. 4

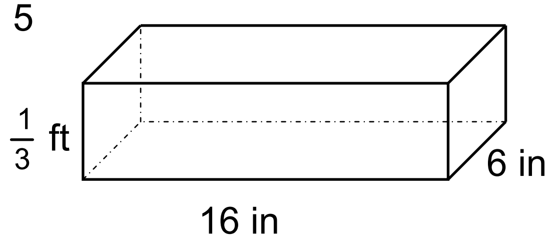



Solution:

The volume of a cube is $s^3$.

Thus, $V = s^3 = (9)^3 = 729 \text{ ft}^3$.

Ex. 5




Solution:

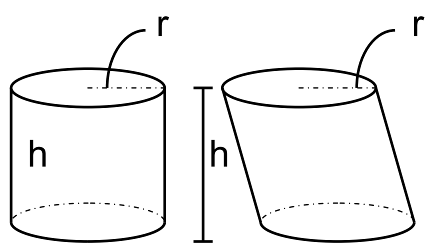
First convert $\frac{1}{3}$ ft into inches:

$\frac{1}{3} = \frac{1\text{ft}}{3} \bullet \frac{12\text{in}}{1\text{ft}} = 4$ in. The volume of a rectangular solid is $V = Lwh$ $= (16)(6)(4) = 384 \text{ in}^3$.

We can think of a cylinder as a type of prism. Its base is a circle so the area of the base is $\pi r^2$ and the height is h. Thus, the volume for a cylinder is $V = Bh = \pi r^2 h$.

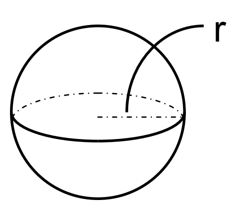
**Volume of a Cylinder**

In general, if r is the radius of the cylinder and h is the height of the cylinder, then the volume is given by $V = \pi r^2 h$.




**Volume of a Sphere**

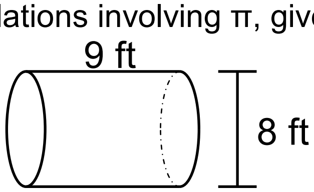
In general, if r is the radius of the sphere, then the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$




**Find the volume of the following:**

(For calculations involving π, give both the exact answer and the approximate answer).

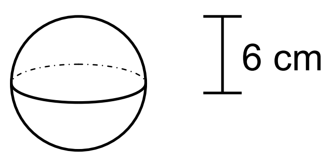
Ex. 6




Solution:

First, we divide the diameter by 2 to get the radius: $8 \div 2 = 4$ ft.

Ex. 7




Solution:

Since the radius is 6 cm, then the volume is $V = \frac{4}{3}\pi r^3$

Think of this as a cylinder lying down on its side. So, the height is 9 ft. Thus, the volume is $V = \pi r^2 h$
$= \pi(4)^2(9) = \pi(16)(9) = 144\pi$ ft$^3$.
$\approx 144(3.14) = 452.16$ ft$^3$.
Exact Answer: $144\pi$ ft$^3$
Approximate: 452.16 ft$^3$

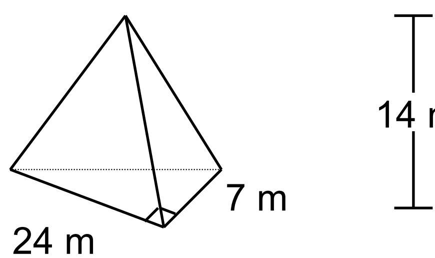
$= \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216)$
$= \frac{4}{3}\pi\left(\frac{216}{1}\right) = \frac{4}{1}\pi\left(\frac{72}{1}\right)$
$= 288\pi$ cm$^3 \approx 288(3.14)$
$= 904.32$ cm$^3$.

Exact Answer: $288\pi$ cm$^3$
Approximate: 904.32 cm$^3$

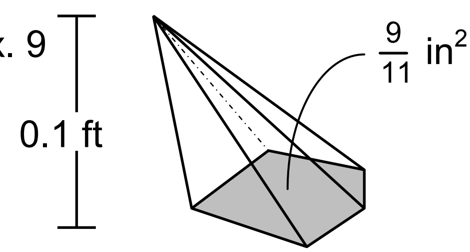
A **Pyramid** is a solid figure with a polygon as a base and sides that are triangles which meet at a common point at the top opposite the base. The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base.

**Find the volume of the following:**

Ex. 8




Ex. 9



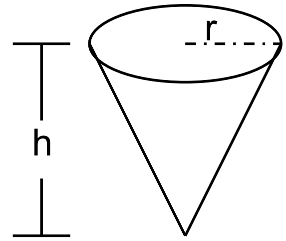

Solution:
First, we need to find the area of the base. Since the base is a triangle with b = 7 m and h = 24, then the area of the triangle is $B = \frac{1}{2}bh = \frac{1}{2}(7)(24)$
$= 84$ m$^2$. The volume of the pyramid is $V = \frac{1}{3}Bh = \frac{1}{3}(84)(14)$
$= 392$ m$^3$.

Solution:
First, we need to convert the 0.1 ft into inches:
$0.1 \text{ ft} = \frac{1\text{ft}}{10} \bullet \frac{12\text{in}}{1\text{ft}} = \frac{6}{5}$ in.
Since $B = \frac{9}{11}$ in$^2$ & $h = \frac{6}{5}$ in, then the volume of the pyramid is $V = \frac{1}{3}Bh$
$= \frac{1}{3}Bh = \frac{1}{3}\left(\frac{9}{11}\right)\left(\frac{6}{5}\right)$
$= \frac{18}{55}$ in$^3$.

If we think of a cone as being like a pyramid with a circular base, then the area of the base is $B = \pi r^2$. So, the volume of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$.

**Volume of a Cone**

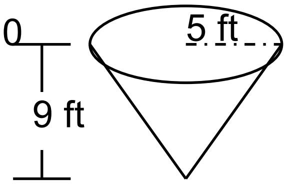
In general, is r is the radius of the cone and h is the height of the cone, then the volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$.




**<u>Find the volume of the following:</u>**

(For calculations involving π, give both the exact answer and the approximate answer).

Ex. 10



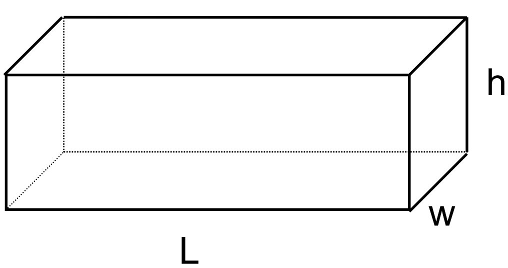

<u>Solution:</u>

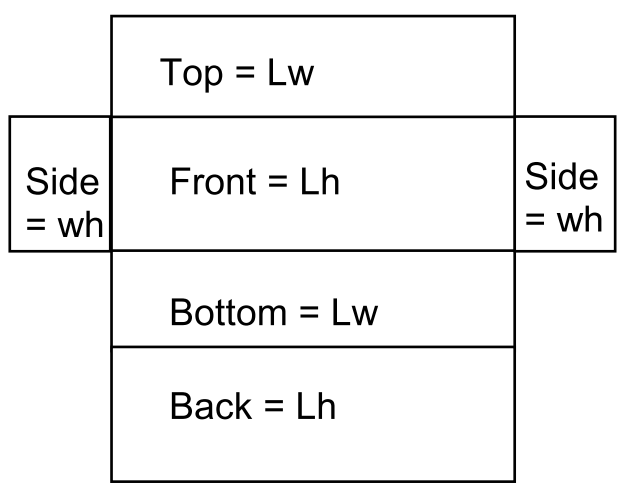
Since h = 9 ft and r = 5 ft, we can plug directly into $V = \frac{1}{3}\pi r^2 h$ to get the answer: $V = \frac{1}{3}\pi(5)^2(9) = \frac{1}{3}\pi(25)(9) = 75\pi \text{ ft}^3$ (Exact answer)

$\approx 75(3.14) = 235.5 \text{ ft}^3$ (Approximate answer).

Objective b: Understanding Surface Area.

The **<u>Surface Area</u>** of a geometric solid is the area on the surface of the solid. Surface area is measured in square units such as $in^2$ or $cm^2$. We use surface area to determine how much aluminum we need to make a soda can or how much wrapping paper we need to wrap a present. If we view a rectangular solid or prism, we see that it consists of six rectangles for its sides:

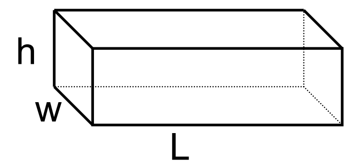



The sum of their areas is Lw + wh + Lh + wh + Lw + Lh. Combining like terms gives us the formula: SA = 2Lw + 2Lh + 2wh

**Surface Area of a Rectangular Solid or Prism**

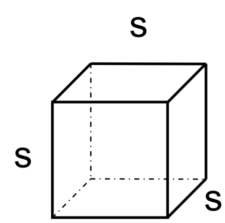
In general, if L is the length, w is the width and h is the height of the rectangular solid, then the formula for the surface area is SA = 2Lw + 2Lh + 2wh.




A cube is a special type of Rectangular Solid where the lengths of all the sides are equal. If the length of each side is equal to s, then L = w = h = s. Hence, the formula for the surface area of a cube becomes:
$2s^2 + 2s^2 + 2s^2 = 6s^2$.

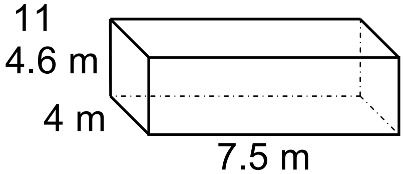
**Surface Area of a Cube**

In general, if s is the length of the side of the cube, then the formula for the surface area of a cube is SA = $6s^2$.



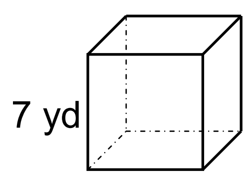

__**Find the surface area of the following:**__

Ex. 11



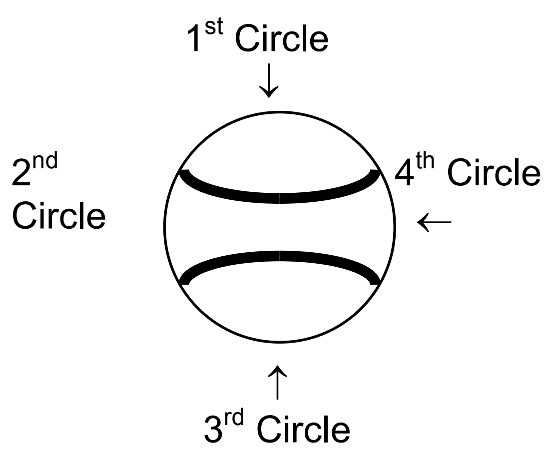

Solution:
For a rectangular prism, the formula for the surface area is
SA = 2Lw + 2Lh + 2wh
= 2(7.5)(4) + 2(7.5)(4.6) + 2(4)(4.6)
= 60 + 69 + 36.8 = 165.8 $m^2$.

Ex. 12



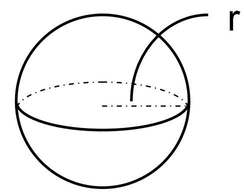

Solution:
For a cube, the formula for the surface area is SA = $6s^2$
= $6(7)^2 = 6(49) = 294$ $yd^2$.

If you look at a softball or a baseball, you can see that it is "roughly covered by four circles." The area of each circle is $\pi r^2$, so the area of "the four circles" is $4\pi r^2$. Thus, for a sphere, the surface area is $4\pi r^2$.

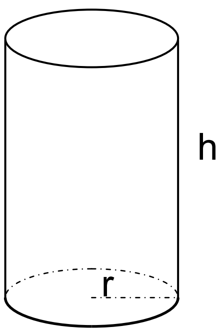


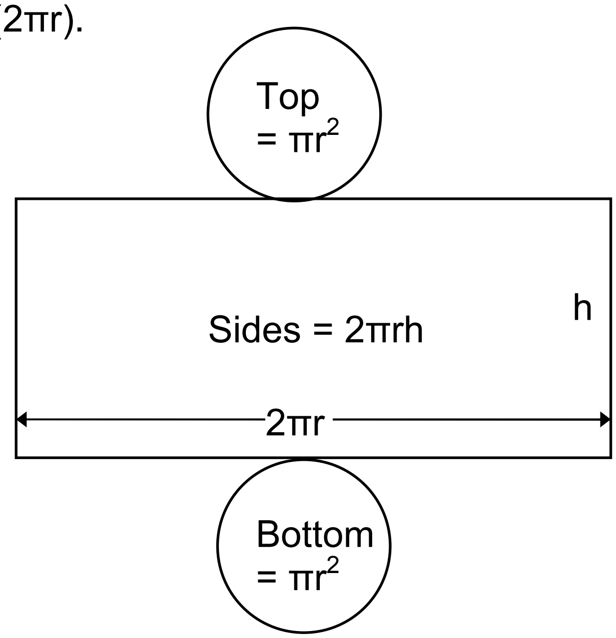
## Surface Area of a Sphere

In general, if r is the radius of the sphere, then the surface area of a sphere is given by $SA = 4\pi r^2$.




For the right cylinder (the height is perpendicular to the base), we can view the top and bottom as two circles and if we cut the side straight down and flatten it out, we will get a rectangle with the width equal to h and the length equal to the circumference of the circle ($2\pi r$).



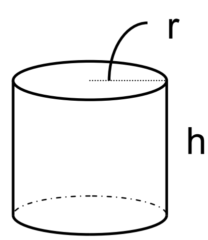




Thus, the area of the side will be $2\pi rh$. Hence, the formula for the surface area of cylinder is

$$SA = \text{Top} + \text{Bottom} + \text{Sides}$$
$$= \pi r^2 + \pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi rh$$

## Surface Area of a Right Cylinder

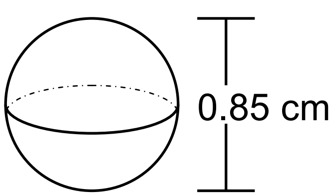
In general, if r is the radius of the right cylinder and h is the height of the right cylinder, then the surface area is given by $SA = 2\pi r^2 + 2\pi rh$.



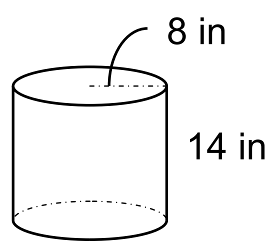

**<u>Find the surface area of the following:</u>**

(For calculations involving π, give both the exact answer and the approximate answer).

Ex. 13

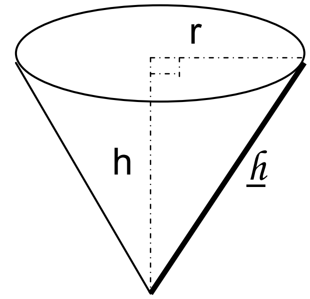



Ex. 14



Solution:
We first need to divide the diameter by two to get the radius: $0.85 \div 2 = 0.425$ cm.
So, the surface area is $SA = 4\pi r^2$
$= 4\pi(0.425)^2 = 4\pi(0.180625)$
$= 0.7225\pi$ cm$^2$ (Exact Answer)
$\approx 2.26865$ cm$^2$ (Approximation)

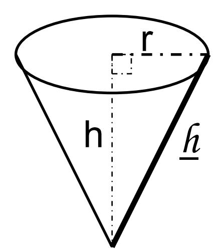
Solution:
The radius is 8 in and 14 in is the height, so the surface area is $SA = 2\pi r^2 + 2\pi rh$
$= 2\pi(8)^2 + 2\pi(8)(14)$
$= 2\pi(64) + 2\pi(8)(14)$
$= 128\pi + 224\pi$
$= 352\pi$ in$^2$ (Exact Answer)
$\approx 1105.28$ in$^2$.

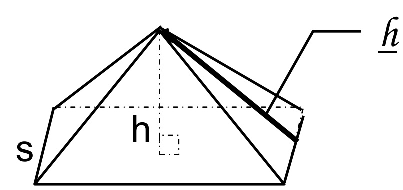



In dealing with the cone, we need to use a new measurement to find its surface area. The distance from the tip of the cone to the edge of the circle is called the <u>slant height</u> of the cone. We will denote the slant height by $\underline{h}$ to distinguish it from the regular vertical height h used in the calculation of the volume of a cone. The top of the cone is a circle with area of $\pi r^2$ whereas the side will have area of $\pi r\underline{h}$ (think of the sides equivalent to a triangle with height of $\underline{h}$ and base equal to circumference of the circle). Therefore, the surface area of a cone is area of the circle plus the area of the equivalent triangle or $SA = \pi r^2 + \pi r\underline{h}$.

**Surface Area of a Cone**

In general, if r is the radius of the cone and $\underline{h}$ is the slant height of the cone, then the surface area of a cone is given by the formula $SA = \pi r^2 + \pi r\underline{h}$.

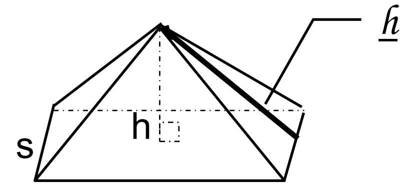



For a pyramid with a square base and where the sides are four congruent triangles, we will define the slant height $\underline{h}$ as altitude of each of the four congruent triangles. The base is a square with the length of each side equal to so its area is $s^2$. Each of the four congruent triangles has a base equal to s and a height equal to the slant height $\underline{h}$, so the area of each of the four triangles is $\frac{1}{2}s\underline{h}$. So, the total area of the sides is $4(\frac{1}{2}s\underline{h}) = 2s\underline{h}$. Hence, the surface area of this pyramid is $SA = s^2 + 2s\underline{h}$.

**Surface Area of a Pyramid with a square base.**

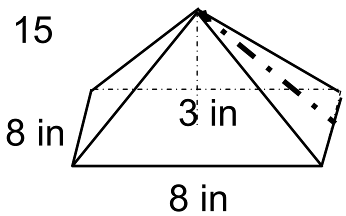
In general, if s is the length of the side of the base and $\underline{\hbar}$ is the slant height, then the formula for the surface area is $SA = s^2 + 2s\underline{\hbar}$.




**<u>Find the surface area of the following:</u>**

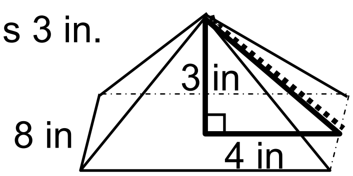
(For calculations involving π, give both the exact answer and the approximate answer).

Ex. 15




Solution:

To solve this problem, we first need to find the slant height. Embedded in the pyramid is a right triangle where the base is half of 8 or 4 in and the height is 3 in.




The hypotenuse of this triangle is the slant height. Using the Pythagorean Theorem, we get:

$c^2 = a^2 + b^2$

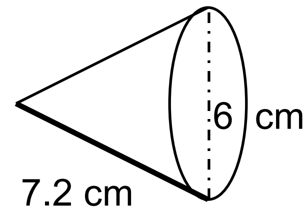
$c^2 = (3)^2 + (4)^2 = 9 + 16$

$c^2 = 25$

$c = \pm\sqrt{25} = \pm 5$

So, $\underline{\hbar} = 5$ in. Plugging into the formula for the surface area, we get: $SA = s^2 + 2s\underline{\hbar} = (8)^2 + 2(8)(5) = 64 + 80 = 144\ in^2$.

Ex. 16




Solution:

To solve this problem, we need to find the radius of the cone by dividing the diameter by 2:

$r = 6 \div 2 = 3$ cm

The formula for the surface area of a cone is

$A = \pi r^2 + \pi r\underline{\hbar}$.

Replacing r by 3 cm and $\underline{\hbar}$ by 7.2 cm, we get:

$A = \pi r^2 + \pi r\underline{\hbar}$

$= \pi(3)^2 + \pi(3)(7.2)$

$= 9\pi + 21.6\pi$

$= 30.6\pi\ cm^2$ (Exact Answer)

$\approx 96.084\ cm^2$ (Approximation)