Sect 7.1 Algebraic Language and Formulas.

Objective 1: Understanding Variables and Expressions

In Algebra, when there is a number that we do not know its value, we represent the number using a **variable** like the letter x. So, if we want to write three times an unknown number, we write 3•x or 3x. An **Algebraic Expression** is a collection of numbers, variables (letters), grouping symbols, and operations (addition "sum", subtraction "difference", multiplication "product", and division "quotient"). Some examples of algebraic expressions are:

Ex. 1a $9y - 6z + 2$ Ex. 1b $\frac{8x^2}{7y} - 3(x + 5)$

Ex. 2a $0.45x^2y - 3xy^2$ Ex. 2b $x^3 - 27y^3$

There are several ways we can represent operations in algebra:

Operation	Algebraic Expression	Translation
Addition	a + b	The sum of a and b.
Subtraction	a – b	The difference of a and b.
Multiplication	a•b, a(b), (a)b, (a)(b), ab (Note a x b is rarely used)	The product of a and b.
Division	$a \div b$, $\frac{a}{b}$, a/b, $b\overline{)a}$	The quotient of a and b.

Parentheses can be indicated by parentheses ( ). Curly brackets { }, or brackets [ ]. In arithmetic, parentheses means "do that first." In Algebra, parentheses means that we treat the quantity inside as a unit. Thus, if we see – 3(x + 5), this means we are multiplying the unit x + 5 by – 3.

Write the following using the appropriate notation:

Ex. 3a The product of 2 and x divided by y.

Ex. 3b The difference of (3x – 2) and (2x – 5).

Ex. 3c The quotient of x^2 and the sum of y and z:

Solution:

a) "The product of 2 and x": $2 \bullet x = 2x$

"divided by y": $2x \div y = \frac{2x}{y}$

b) "The difference" means subtract:

$(3x - 2) - (2x - 5)$

c) "The sum of y and z": $(y + z)$

"The quotient of x^2 and … ": $x^2 \div (y + z) = \frac{x^2}{y+z}$

In arithmetic, a **factor** is a number that divides a quantity evenly with no remainder. Thus, 3 and 5 are factors of 15 since they both divide 15 evenly with no remainder, but 7 is not a factor of 15 since 7 divides 15 with a remainder of 1. In Algebra, the product (or quotient) of factors form a quantity or **term**. Hence, in the quantity $4xyz(a + b)$, the factors are 4, x, y, z, and $(a + b)$.

Find the factors of the following:

Ex. 4a $-3ab^2c$

Ex. 4b $19(c - d)(r + h)$

Ex. 4c $\frac{6a^2b}{c^3}$

Solution:

a) The factors of $-3ab^2c$ are -3, a, b (two), and c

b) The factors of $19(c - d)(r + h)$ are 19, $(c - d)$, and $(r + h)$.

c) The factors of $\frac{6a^2b}{c^3}$ are 6, a (two), b, and c (three).

We say a **term** is a number, a letter, or the product and/or quotient of numbers and letters. Notice that there is no mention of addition or subtraction in this definition. When we look at an expression, we can look for addition and subtraction to determine where one term ends and another one begins.

Determine the number of terms in the following expressions:

Ex. 5a $-0.3a^2 + 4.2b^2$

Solution:

$-0.3a^2$ is the first term and $4.2b^2$ is the second term, so $-0.3a^2 + 4.2b^2$ has two terms.

Ex. 5b $3x^2 - 6xy + 5$

Solution:

$3x^2$ is the first term, $-6xy$ is the second, and 5 is the third, so $3x^2 - 6xy + 5$ has three terms.

Ex. 6a $-\frac{5}{6}r^2s + \frac{2}{3}rs^2 - \frac{11}{9}$

Solution:

The terms are $-\frac{5}{6}r^2s$, $\frac{2}{3}rs^2$, and $-\frac{11}{9}$, so the expression has three terms.

Ex. 6b $-\frac{7a^4b^5c}{8x^2y}$

Solution:

Since $-\frac{7a^4b^5c}{8x^2y}$ is the only term, then $-\frac{7a^4b^5c}{8x^2y}$ has one term.

Fill in the blank:

Ex. 7a $7x^2 - 8yz$ $7x^2$ is a ____________

Ex. 7b $5fg + 14g^2$ f is a ______________

Ex. 7c $3x(a - 7t)$ 3x is a _____________

Ex. 7d $3x - (a - 7t)$ 3x is a _____________

Ex. 7e $-2x^2 + 11yz^2$ $11yz^2$ is a __________

Solution:

a) $7x^2$ is a term.
b) f is a factor.
c) 3x is a factor.
d) 3x is a term.
e) $11yz^2$ is a term.

Objective 2: Evaluating Expressions and Formulas.

To evaluate an expression, replace the letters by the numbers given. Use a set of parenthesis around the number when you plug-in the number.

Evaluate the following for x = 0.3 and y = 1.3:

Ex. 8a $x + y$

Ex. 8b $5.0x - y$

Ex. 8c xy

Solution:

a) $x + y = (0.3) + (1.3) = 1.6$
b) $5.0x - y = 5.0(0.3) - (1.3) = 1.5 - 1.3 = 0.2$
c) $xy = (0.3)(1.3) = 0.39 \approx 0.4$

Evaluating formulas works the same way as evaluating expressions except the value of the expression is a specific variable.

Evaluate the following formulas:

Ex. 9a d = 55t for t = 4.0 hr
Ex. 9b V = IR for I = 5 amps and R = 12 ohms
Ex. 9c Pe = Te – Ke for Te = 1220 J and Ke = 350 J

Solution:
a) d = 55(4) = 220 miles.
b) V = (5)(12) = 60 ohms
c) Pe = (1220) – (350) = 870 J

For more complicated problems, we need to remember the order of operations:

Order of Operations

1) Parentheses - Simplify the expression inside of Parentheses or grouping symbol: ( ), [ ], { }, $| \; |$, $\sqrt{\;\;}$
2) Exponents including square roots.
3) Multiplication or Division as they appear from left to right.
4) Addition or Subtraction as they appear from left to right.

A common phrase people like to use is:

Please
Excuse
My **D**ear
Aunt **S**ally

(Be careful with the My Dear and the Aunt Sally part. Multiplication does not precede Division and Division does not precede multiplication; they are done as they appear from left to right. The same is true for addition and subtraction.)

Evaluate each formula using the given values:

Ex. 10 A = p + prt
p = \$550, r = 0.0800, & t = 0.750

Solution:
A = p + prt
A = (550) + (550)(0.08)(0.75)
A = 550 + 33 = \$583
A = \$583

Ex. 11 $A = 0.5(b_1 + b_2)h$
b_1 = 9 ft, b_2 = 16 ft, & h = 4 ft

Solution:
$A = 0.5(b_1 + b_2)h$
A = 0.5([9] + [16])(4)
A = 0.5(25)(4)
A = 50 ft^2

Ex. 12 $F = \frac{9}{5}C + 32$; $C = -25^\circ$

<u>Solution:</u>

$F = \frac{9}{5}C + 32$

$F = \frac{9}{5}(-25) + 32$

$F = -45 + 32$

$F = -13^\circ$ F

Ex. 13 $d = -16t^2 + 48t$; $t = 4.0$ s

<u>Solution:</u>

$d = -16t^2 + 48t$

$d = -16(4)^2 + 48(4)$

$d = -16(16) + 48(4)$

$d = -256 + 192$

$d = -64$ ft

Ex. 14 $P = \frac{nRT}{V}$; n = 9.0 moles, R = 0.2502, T = 48.0° F, V = 23.0 ft^3

<u>Solution:</u>

$P = \frac{nRT}{V}$

$P = \frac{(9)(0.2502)(48)}{(23)}$

$P = \frac{108.0864}{23}$

$P = 4.6994\ldots \approx 4.7$ psi

Ex. 15 $t = 2.0\,\pi\sqrt{\frac{L}{9.8}}$; $\pi \approx 3.14$, L = 0.80 m

<u>Solution:</u>

$t = 2.0\,\pi\sqrt{\frac{L}{9.8}}$

$t = 2.0(3.14)\sqrt{\frac{0.8}{9.8}}$

$t = 2.0(3.14)(0.2857\ldots)$

$t = 1.79428\ldots$

$t \approx 1.8$ sec

Solve the following:

Ex. 16 Gary is having nerve trouble with his arm. To test the electrical conduction of his nerves, a doctor measures the length from Gary's elbow to his finger tip and finds it to be 0.4318 meters. The doctor then measures the time between the initiation pulse at Gary's elbow and the detection of the pulse at Gary's fingertip and finds that to be 0.01925 seconds.

a) Use $r = \frac{d}{t}$ to calculate the speed of the pulse.

b) If the normal range is between 50 and 60 meters per second, was the speed of the pulse through Gary's arm in the normal range?

Solution:

a) $r = \frac{d}{t}$

$r = \frac{0.4318}{0.01925}$

$r = 22.4311\ldots$

$r \approx 22.43$ m/sec

b) This is below normal.

Ex. 17 The volume of a round steel bar is given by the formula $V = \pi d^2L/4$ where d is the diameter of the rod and L is the length of the rod. Find the volume of a rod that is 28.0 in. long and has a diameter of 1.50 in. Use $\pi \approx 3.14$.

Solution:

$V = \pi d^2L/4$

$V = (3.14)(1.5)^2(28)/4$ (exponents)

$V = (3.14)(2.25)(28)/4$ (multiply)

$V = 197.82/4$ (divide)

$V = 49.455$

$V \approx 49.5 \text{ in}^3$

Ex. 18 For three resistors in parallel, the total resistance R_T is given by:

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_1R_3}.$$

If the three resistors are 1250 ohms, 352 ohms, and 675 ohms, find the total resistance.

Solution:

$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$

$R_T = \frac{(1250)(352)(675)}{(1250)(352) + (352)(675) + (1250)(675)}$ (multiply)

$R_T = \frac{297000000}{440000 + 237600 + 843750}$ (add)

$R_T = \frac{297000000}{1521350}$ (divide)

R_T = 195.2213…≈ 195 ohms

Ex. 19 If a cylindrical container is on its side, the volume of the liquid in the container (in gallons) is given by:

$V = L\left\{Ar^2 - \sqrt{2rh - h^2}\,(r - h)\right\}/231$

Find the volume of the water in the container if it is 52.0 in long (L), has a radius of 47.5 in (r), the water height (h) is 11.875 in, and the angle measure (A) is 0.7227.

Solution:

$V = L\left\{Ar^2 - \sqrt{2rh - h^2}\,(r - h)\right\}/231$

$V = (52)\left\{0.7227(47.5)^2 - \sqrt{2(47.5)(11.875) - (11.875)^2}\,(47.5 - 11.875)\right\}/231$

(exponents under the square root)

$V = (52)\left\{0.7227(47.5)^2 - \sqrt{2(47.5)(11.875) - 141.015625}\,(47.5 - 11.875)\right\}/231$

(multiplication under the square root)

$V = (52)\left\{0.7227(47.5)^2 - \sqrt{1128.125 - 141.015625}\,(47.5 - 11.875)\right\}/231$

(subtraction under the square root and in the parenthesis)

$V = (52)\left\{0.7227(47.5)^2 - \sqrt{987.109375}\,(35.625)\right\}/231$

(exponents and square root in the curly brackets)

$V = (52)\left\{0.7227(2256.25) - (31.41829682)(35.625)\right\}/231$

(multiplication in the curly brackets}

$V = (52)\left\{1630.591875 - 1119.276824\right\}/231$

(subtraction in the curly bracket)

$V = (52)\left\{511.3150508\right\}/231$ (multiplication & division)

V = 115.101… ≈ 115 gallons