Sect 7.2 – Adding & Subtracting Algebraic Expressions

Objective 1: Understanding Numerical Coefficient

The **numerical coefficient** of a term is the numerical (constant) factor of a term. In – 6xy, the numerical coefficient is – 6. Let's try some examples:

Determine the numerical coefficient of the indicated term:

Ex. 1a $-0.3a^2 + 4.2b^2$; 1st term

Ex. 1b $-0.3a^2 + 4.2b^2$; 2nd term

Ex. 1c $14x - 17y + 6$; 2nd term

Ex. 1d $9x^2 - 4xy - y^3$; 3rd term

Ex. 1e $\frac{19}{6}y^2 + y - 7$; 2nd term

Ex. 1f $-\frac{7a^4b^5c}{8x^2y}$; 1st term

Ex. 1g $9x^2 - 7xy + 8$; 3rd term

Solution:

a) The numerical coefficient of $-0.3a^2$ is – 0.3.
b) The numerical coefficient of $4.2b^2$ is 4.2.
c) The numerical coefficient of – 17y is – 17.
d) The numerical coefficient of $-y^3$ is – 1.
e) The numerical coefficient of y is 1.
f) The numerical coefficient of $-\frac{7a^4b^5c}{8x^2y}$ is $-\frac{7}{8}$.
g) The numerical coefficient of 8 is 8.

Objective 2: Understanding How to Combine Like Terms

In terms of simplifying variable expressions, we first need to examine what happens in the real world when you combine items:

Simplify the following:

Ex. 2a 3 apples + 4 apples

Ex. 2b 3 apples + 4 oranges

Solution:

a) 3 apples + 4 apples = 7 apples.
b) 3 apples + 4 oranges = 3 apples + 4 oranges.

In example 2a, we are able to add the items together since the units (apples) are the same whereas in example 2b, we cannot put the items together since the units (apples and oranges) are not the same. It would not make any sense to say 7 appleoranges for the answer to 2b since there is no such thing as an appleorange (or at least, scientists have not invented one yet!). This same idea applies to algebra:

<u>Simplify the following:</u>

Ex. 3a $3x^2 + 4x^2$ Ex. 3b $3x^2 + 4y$

<u>Solution:</u>

a) Instead of having apples, we have x^2's so $3x^2 + 4x^2 = 7x^2$.

b) This is exactly like adding apples and oranges so, $3x^2 + 4y = 3x^2 + 4y$. It cannot be simplified any further.

In example 3a, we are able to add the items together since the "units" (x^2) are the same whereas in example 3b, we cannot since the "units" (x^2 and y) are different. When terms have exactly the same "units", they are called **<u>like terms</u>**. Here is a more precise definition:

Terms with exactly the same variables with exactly the same corresponding exponents are called **<u>like terms</u>**. Only like terms can be combined by combining their numerical coefficients.

<u>Simplify the following:</u>

Ex. 4 $-3x + 5 + 7x - 3$

<u>Solution:</u>

We have two sets of like terms; the first pair is – 3x and 7x and the second pair is 5 and – 3. We will combine each pair by adding their numerical coefficients:

$\underline{-3x} \mathbf{+ 5} \underline{+7x} \mathbf{- 3} = \underline{-3x}\ \underline{+7x}\ \mathbf{+ 5 - 3} = 4x + 2$

Ex. 5 $0.8x^2 - 6xy + 1.2y^2 - 1.9x^2 - 7xy + 3.4y^2$

<u>Solution:</u>

First identify are sets of like terms and then combine:

$\mathbf{0.8x^2} - 6xy \underline{+ 1.2y^2} \mathbf{- 1.9x^2} - 7xy \underline{+ 3.4y^2}$

$= \mathbf{0.8x^2 - 1.9x^2} - 6xy - 7xy \underline{+ 1.2y^2 + 3.4y^2} = \mathbf{- 1.1x^2} - 13xy \underline{+ 4.6y^2}$

$= -1.1x^2 - 13xy + 4.6y^2$.

Ex. 6 $-\frac{4}{3}x + \frac{2}{3}xy^2 - \frac{13}{8}x^2 - \frac{5}{6}x + \frac{9}{7}x^2y$

<u>Solution:</u>

Since $-\frac{4}{3}x$ and $-\frac{5}{6}x$ are the only like terms, they are the only ones that can be combined. Note that $\frac{2}{3}xy^2$ and $\frac{9}{7}x^2y$ are not like terms since the corresponding exponents are not the same (the power of x in $\frac{2}{3}xy^2$ is one whereas the power of x in $\frac{9}{7}x^2y$ is two.

Since $-\frac{4}{3} - \frac{5}{6} = -\frac{13}{6}$, then

$\underline{-\frac{4}{3}\mathbf{x}} + \frac{2}{3}xy^2 - \frac{13}{8}x^2 \underline{-\frac{5}{6}\mathbf{x}} + \frac{9}{7}x^2y = \mathbf{-}\frac{13}{6}\mathbf{x} + \frac{2}{3}xy^2 - \frac{13}{8}x^2 + \frac{9}{7}x^2y.$

Ex. 7 $5x^2 - 6xy + y^2 + 5y$

Solution:

There are no like terms. This expression cannot be simplified any further. Our answer is $5x^2 - 6xy + y^2 + 5y$.

Objective 3: Understanding and Applying the Distributive Property.

Let's look at an example that illustrates the distributive property.

Find the following:

Ex. 8 7(3 + 5)

Solution:

Normally, we would add the three and the five together first and then multiply:

7(**3 + 5**) = 7(8) = 56

But, the distributive property says that we can multiply the 3 and 5 both by seven first and then add the results:

7(3 + 5) = **7**•3 + **7**•5 = 21 + 35 = 56.

Notice that we get the same result either way. This is an extremely important property in mathematics.

Distributive Property

If a, b, c, and d are real numbers, then

a(b + c + d) = **a**•b + **a**•c + **a**•d and **a**(b – c – d) = **a**•b – **a**•c – **a**•d

Let's see how this applied in algebra.

Simplify:

Ex. 9 $7(3x + 5)$

Solution:

Here, we cannot combine the 3x with the 5 since they are not like terms, but we can use the distributive property:

$\mathbf{7}(3x + 5) = \mathbf{7}\bullet 3x + \mathbf{7}\bullet 5 = 21x + 35$.

Ex. 10 $-2.2(3q - 4)$

Solution:

$\mathbf{-2.2}(3q - 4) = \mathbf{-2.2}\bullet 3q - (\mathbf{-2.2})\bullet 4 = -6.6q - (-8.8) = -6.6q + 8.8$.

Ex. 11 $(2x - 4y + 5)(5)$

Solution:

$(2x - 4y + 5)\mathbf{(5)} = 2x\mathbf{(5)} - 4y\mathbf{(5)} + 5\mathbf{(5)} = 10x - 20y + 25$

Ex. 12 $-4(-3x + 6 - 2y)$

Solution:

$\mathbf{-4}(-3x + 6 - 2y) = \mathbf{-4}(-3x) + (\mathbf{-4})\bullet 6 - (\mathbf{-4})\bullet 2y$

$= 12x + (-24) + 8y$, but, in algebra, we want to use the least number of symbols possible to express our answer, so we will rewrite the answer as $12x - 24 + 8y$.

Ex. 13 $r(3r - 4)$

Solution:

$\mathbf{r}(3r - 4) = \mathbf{r}\bullet 3r - r\bullet 4 = 3r^2 - 4r$

Ex. 14 $-\frac{4}{7}(28x^2 - 35x)$

Solution:

$-\frac{4}{7}(28x^2 - 35x) = \left(-\frac{4}{7}\right)(28x^2) - \left(-\frac{4}{7}\right)(35x)$

$= -16x^2 + 20x$

Ex. 15 $-(-11 + 3p - p^2)$

Solution:

$-(-11 + 3p - p^2) = \mathbf{-1}(-11 + 3p - p^2)$

$= \mathbf{(-1)}(-11) + \mathbf{(-1)}(3p) - \mathbf{(-1)}(p^2) = 11 + (-3p) - (-p^2)$

$= 11 - 3p + p^2$

Ex. 16 $4.1(3x - 2) - 5.2(2x + 8)$

Solution:

Distribute first and then combine like terms (multiplication comes before adding and subtracting):

$\mathbf{4.1}(3x - 2) \mathbf{- 5.2}(2x + 8) = \mathbf{4.1}(3x) - \mathbf{4.1}(2) \mathbf{- 5.2}(2x) + \mathbf{- 5.2}(8)$

$= \mathit{12.3x} \underline{- 8.2} - \mathit{10.4x} \underline{- 41.6} = \mathit{12.3x} - \mathit{10.4x} \underline{- 8.2 - 41.6}$

$= 1.9x - 49.8$

Ex. 17 $y(3y - 8) - 2(y^2 + 6)$

Solution:

Distribute first and then combine like terms:

$y(3y - 8) - 2(y^2 + 6) = \mathbf{y}(3y - 8) \mathbf{- 2}(y^2 + 6)$

$= \mathbf{y}(3y) - \mathbf{y}(8) - \mathbf{2}(y^2) + (\mathbf{- 2})(6) = \underline{3y^2} - 8y - \underline{2y^2} - 12$

$= 1y^2 - 8y - 12 = y^2 - 8y - 12$

Ex. 18 $\frac{2}{3}(5x - \frac{9}{2}) - \frac{3}{13}(39x + \frac{1}{7})$

Solution:

Distribute first and then combine like terms:

$\frac{2}{3}(5x - \frac{9}{2}) - \frac{3}{13}(39x + \frac{1}{7}) = \frac{2}{3}\left(5x\right) - \frac{2}{3}\left(\frac{9}{2}\right) - \frac{3}{13}\left(39x\right) - \frac{3}{13}\left(\frac{1}{7}\right)$

$= \frac{10x}{3} - 3 - 9x - \frac{3}{91} = \frac{10x}{3} - 9x - 3 - \frac{3}{91}$

$= -\frac{17}{3}x - \frac{276}{91}$