Sect 7.5 – Solving More Equations and Formulas

Objective 1: Steps to Solve a Linear Equation in One Variable.

When working with equations, we may need to simplify each side of the equation before solving. This may include:

i) Using the Distributive Property to Clear Parentheses
ii) Clearing Fractions and Decimals
iii) Combining Like Terms

Thus, we can outline a procedure for solving linear equations:

Procedure for Solving Equations:

1) Simplify each side of the equation. This includes:
   i) Using the Distributive Property to Clear Parentheses
   ii) Clearing Fractions and Decimals
   iii) Combining Like Term
2) Use the addition and subtraction properties of equality to isolate the variables terms on one side, the constant terms on the other side.
3) Use the multiplication and division properties of equality to make the coefficient of the variable term equal to one.
4) Check the solution by plugging in your answer into the original equation and simplifying each side to see if you have the same number on both sides.

Solve the following:

Ex. 1 $-3x + 4 + 5x = 2x - (3x + 8)$

Solution:

First, let us simplify the left side:

$-3x + 4 + 5x$ (combine like terms)

$= 2x + 4$

Now, let us simplify the right side:

$2x - (3x + 8)$ (distribute {"clear parentheses"})

$2x \mathbf{-1}(3x) + \mathbf{(-1)}(8)$ (multiply)

$2x - 3x - 8$ (combine like terms)

$= -x - 8$

Thus, our equation becomes:

$2x + 4 = -x - 8$

Now, we solve the equation:

$$\begin{aligned} 2x + 4 &= -x - 8 \quad &\text{(add x to both sides)}\\ +x \quad &= +x \\ \hline 3x + 4 &= -8 \quad &\text{(subtract 4 from both sides)}\\ -4 &= -4 \\ \hline 3x &= -12 \end{aligned}$$

$$\frac{3x}{3} = \frac{-12}{3} \qquad \text{(divide both sides by 3)}$$

$$x = -4$$

Ex. 2 $\quad 4 - 3(2x - 5) = \frac{1}{8}x - 11 + \frac{1}{4}x$

<u>Solution:</u>

First, let us simplify the left side:

$4 - 3(2x - 5)$ (distribute {“clear parentheses”})

$= 4 \mathbf{- 3}(2x) - (\mathbf{- 3})(5)$ (multiply)

$= 4 - 6x + 15$ (combine like terms)

$= -6x + 19$

Now let us simplify the right side:

$\frac{1}{8}x - 11 + \frac{1}{4}x$ (combine like terms)

$= \frac{3}{8}x - 11$

Thus, our equation becomes:

$$\begin{aligned} -6x + 19 &= \frac{3}{8}x - 11 \quad &\text{(add 6x to both sides)}\\ +6x \quad &= +6x \\ \hline 19 &= \frac{51}{8}x - 11 \quad &\text{(add 11 to both sides)}\\ +11 &= \quad +11 \\ \hline 30 &= \frac{51}{8}x \end{aligned}$$

Now, multiply by the reciprocal of $\frac{51}{8}$ which is $\frac{8}{51}$.

$$\frac{8}{51}\left(30\right) = \frac{8}{51}\left(\frac{51}{8}x\right) \qquad \text{(multiply)}$$

$$\frac{80}{17} = x$$

Ex. 3 $3(2.2x - 6.1) - 4.5x = 7.3x - 6.2 + 11.1x$

Solution:

First, let us simplify the left side:

$3(2.2x - 6.1) - 4.5x$ (distribute)

$= \mathbf{3}(2.2x) - \mathbf{3}(6.1) - 4.5x$ (multiply)

$= 6.6x - 18.3 - 4.5x$ (combine like terms)

$= 2.1x - 18.3$

Now, let us simplify the right side:

$7.3x - 6.2 + 11.1x$ (combine like terms)

$= 18.4x - 6.2$

Thus, our equation becomes:

$2.1x - 18.3 = 18.4x - 6.2$ (subtract 18.4x from both sides)

$\underline{-18.4x \quad = -18.4x}$

$-16.3x - 18.3 = -6.2$ (add 18.3 to both sides)

$\underline{\quad + 18.3 = +18.3}$

$-16.3x = 12.1$

$\frac{-16.3x}{-16.3} = \frac{12.1}{-16.3}$ (divide both sides by – 16.3)

(slide the decimal point over on place to write as a fraction)

$x = -\frac{121}{163}$

Ex. 4 $3x - 8 = -4(2x + 5) - 3(4x - 8)$

Solution:

The left side is already simplified, so we need to focus only on the right side:

$-4(2x + 5) - 3(4x - 8)$ (distribute)

$= \mathbf{-4}(2x) + \mathbf{-4}(5) \mathbf{-3}(4x) - \mathbf{-3}(8)$ (multiply)

$= -8x - 20 - 12x + 24$ (combine like terms)

$= -20x + 4$

Thus, our equation becomes:

$3x - 8 = -20x + 4$ (add 20x to both sides)

$\underline{+20x \;= +20x}$

$23x - 8 = 4$ (add 8 to both sides)

$\underline{\quad +8 = +8}$

$23x = 12$

Now, divide by 23

$\frac{23x}{23} = \frac{12}{23}$

$x = \frac{12}{23}$

Objective 2: Solving Literal Equations for a particular variable.

Now, we will examine solving formulas for a particular variable. Sometimes it is useful to take a formula that is solved for one variable and rewrite it so it is solved for another variable. To see how this is done, we will first work a problem with numbers that we can plug in and solve. We will then use that as a blueprint for solving a formula (or literal equation) for a particular variable.

Solve:

Ex. 5a $A = Lw$ (area of a rectangle); $A = 45\text{ ft}^2$ and $w = 9$ ft

Solution:

Substitute the given values for A and w and solve:

$A = Lw$

$45 = L(9)$ (rewrite the right side)

$\frac{45}{9} = \frac{9L}{9}$ (divide both sides by the number in front of L)

$5 = L$ So, L = 5 ft.

Ex. 5b $A = Lw$ for L

Solution:

Follow the steps we performed in Ex. 5a after substituting:

$A = Lw$ (rewrite the right side)

$\frac{A}{w} = \frac{wL}{w}$ (divide both sides by the number in front of L)

$\frac{A}{w} = L$ So, $L = \frac{A}{w}$.

Ex. 6a $i = prt$ (simple interest); $r = 0.09$, $t = 1.5$ & i = \$945

Solution:

Substitute the given values and solve:

$i = prt$

$945 = p(0.09)(1.5)$ (multiply)

$\frac{945}{0.135} = \frac{0.135p}{0.135}$ (divide by the number in front of p)

$7000 = p$ So, p = \$7000

Ex. 6b $i = prt$ for p

Solution:

Follow the steps we performed in Ex. 6a after substituting:

$i = prt$ (multiply)

$\frac{i}{(rt)} = \frac{(rt)p}{(rt)}$ (divide by the number in front of p)

$\frac{i}{rt} = p$ So, $p = \frac{i}{rt}$

Ex. 7a $T_e = K_e + P_e$ (Total Energy); $P_e = 9152$ J and $T_e = 15612$ J.

Solution:

Substitute the given values and solve:

$T_e = K_e + P_e$

$15612 = K_e + 9152$ (subtract the constant term from both sides)

$15612 = K_e + 9152$

$\underline{-9152 = -9152}$

$6460 = K_e$ So, $K_e = 6460$ J

Ex. 7b $T_e = K_e + P_e$ for K_e

Solution:

Follow the steps we performed in Ex. 7a after substituting:

$T_e = K_e + P_e$ (subtract the constant term from both sides)

$T_e = K_e + P_e$

$\underline{-P_e = -P_e}$

$T_e - P_e = K_e$ So, $K_e = T_e - P_e$

Ex. 8a $P = \frac{F}{A}$ (pressure); $A = 33\ m^2$ and $P = 202{,}000\ N/m^2$

Solution:

Substitute the given values and solve:

$P = \frac{F}{A}$

$202{,}000 = \frac{F}{33}$ (multiply both sides by the

$33(202{,}000) = \frac{F}{33}(33)$ number below F)

$6{,}666{,}000 = F$ So, $F \approx 6{,}700{,}000$ N

Ex. 8b $P = \frac{F}{A}$ for F.

<u>Solution:</u>

Follow the steps we performed in Ex. 8a after substituting:

$P = \frac{F}{A}$ (multiply both sides by the number

$A(P) = \frac{F}{A}(A)$ below F)

$AP = F$ So, $F = AP$

Ex. 9 $V = \pi r^2 h$ for h (volume of the cylinder)

<u>Solution:</u>

Since h is multiplied by πr^2, we need to divide both sides by πr^2:

$$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

So, $h = \frac{V}{\pi r^2}$

Ex.10 $x = a - y$ for a

<u>Solution:</u>

Add y to both sides:

$$\begin{array}{r} x = a - y \\ \underline{+ y = + y} \\ x + y = a \end{array}$$

So, $a = x + y$

Ex. 11 $R = \frac{V}{I}$ for V (Ohm's Law)

<u>Solution:</u>

Multiply both sides by I to solve for V:

$R = \frac{V}{I}$

$I(R) = I\left(\frac{V}{I}\right)$

$IR = V$ So, $V = IR$

Ex. 12 $R_s = R_1 + R_2 + R_3$ for R_1 (resistors in series)

<u>Solution:</u>

Isolate the term with R_1 by itself and solve:

$$\begin{array}{r} R_s = R_1 + R_2 + R_3 \\ \underline{- R_2 - R_3 = \quad - R_2 - R_3} \\ R_s - R_2 - R_3 = R_1 \end{array}$$

(subtract $R_2 + R_3$ from both sides)

So, $R_1 = R_s - R_2 - R_3$

Ex. 13 $A = P(1 + rt)$ for r

Solution:

This formula is more complicated then the others we have solved so far. To see how to do this problem, let us pretend for the moment that A = 7, P = 5, and t = 3 and see how we would solve that equation:

$A = P(1 + rt)$ (plug in the numbers)

$7 = 5(1 + r(3))$ (distribute the 5)

$7 = 5 + 15r$ (subtract 5 from both sides)

$\underline{-5 = -5}$

$2 = 15r$

$\frac{2}{15} = \frac{15r}{15}$ (divide both sides by 15)

$r = \frac{2}{15}$

Now, let us follow the same steps to solve $A = P(1 + rt)$ for r:

$A = P(1 + \mathbf{r}t)$ (distribute the P)

$A = P + (Pt)\mathbf{r}$ (subtract P from both sides)

$\underline{-P = -P}$

$A - P = (Pt)\mathbf{r}$

$\frac{A - P}{Pt} = \frac{(Pt)\mathbf{r}}{Pt}$ (divide both sides by Pt)

$r = \frac{A - P}{Pt}$

Ex. 14 Solve $i = \frac{V}{R_1 + R_2}$ for R_2. (current through two resistors in series)

Solution:

$i = \frac{V}{R_1 + R_2}$ (multiply both sides by $(R_1 + R_2)$)

$i\mathbf{(R_1 + R_2)} = \frac{V}{R_1 + R_2}\mathbf{(R_1 + R_2)}$ (simplify)

$i(R_1 + R_2) = V$ (distribute)

$iR_1 + iR_2 = V$ (subtract iR_1)

$\underline{-iR_1 \quad = \quad -iR_1}$

$\frac{iR_2}{I} = \frac{V - iR_1}{i}$ (divide by i)

$R_2 = \frac{V - iR_1}{i}$

Note, when dividing an expression by a number, we must divide each term by that number. Thus, $\frac{V - iR_1}{i} \neq V - R_1$ since we must also divide V by i.

Ex. 15 Solve $F = \frac{9}{5}C + 32$ for C

Solution:

$$F = \frac{9}{5}\mathbf{C} + 32 \quad \text{(multiply both sides by 5 to clear fractions)}$$

$$5(F) = 5\left(\frac{9}{5}\mathbf{C} + 32\right) \quad \text{(distribute)}$$

$$5F = 5\left(\frac{9}{5}\mathbf{C}\right) + 5(32) \quad \text{(multiply)}$$

$$5F = 9\mathbf{C} + 160 \quad \text{(subtract 160 from both sides)}$$

$$\underline{-160 = \quad -160}$$

$$5F - 160 = 9\mathbf{C}$$

$$\frac{5F - 160}{9} = \frac{9\mathbf{C}}{9} \quad \text{(divide both sides by 9)}$$

$$C = \frac{5F-160}{9} \text{ or } C = \frac{5(F-32)}{9} \quad \text{(since } 160 \div 5 = 32\text{)}$$

$$\text{or} \quad C = \frac{5}{9}(F - 32)$$

Ex. 16 Solve P = 2L + 2w for w

Solution:

This is a formula for the perimeter of a rectangle.

$$P = 2L + 2\mathbf{w} \quad \text{(subtract 2L from both sides)}$$

$$\underline{-2L = -2L}$$

$$P - 2L = 2\mathbf{w}$$

$$\frac{P - 2L}{2} = \frac{2\mathbf{w}}{2} \quad \text{(divide both sides by 2)}$$

$$w = \frac{P-2L}{2} \quad \text{or} \quad w = \frac{P}{2} - L$$