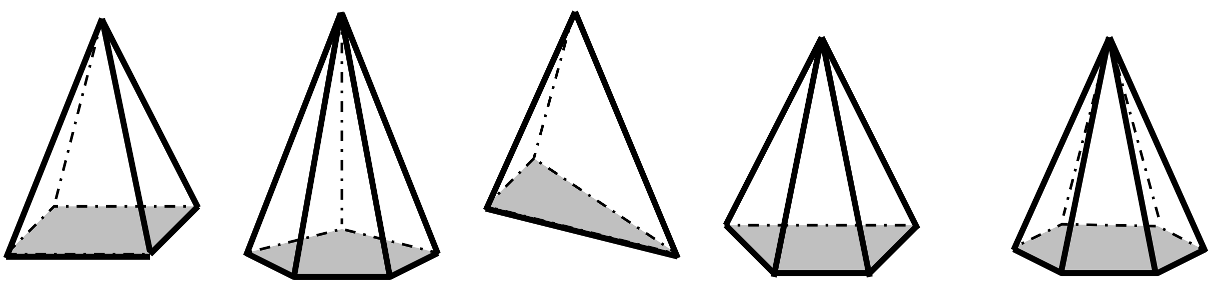
Sect 9.2 - Pyramids

Objective 1 Understanding Pyramids, Volumes, and Surface Area.

A **Pyramid** is a solid figure with a polygon as a base and triangular faces that meet at a common point (apex) on the opposite end of the base. The perpendicular distance between the base and the point is called the height or altitude. In this course, we will only consider pyramids where the base is a regular polygon and apex is centered over the base. We call these types of pyramids "right pyramids." The slant height is the height of one of the triangular faces. In each pyramid below, the base is shaded in gray.



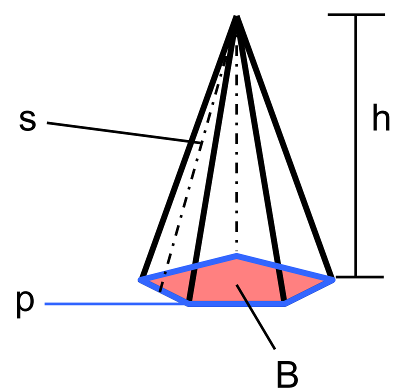
Rectangular Pyramid | Pentagonal Pyramid | Triangular Pyramid | Trapezoidal Pyramid | Hexagonal Pyramid

Properties of Right Pyramids:

Let p = the perimeter of the base
B = the area of the base
h = the height of the Pyramid
s = the slant height

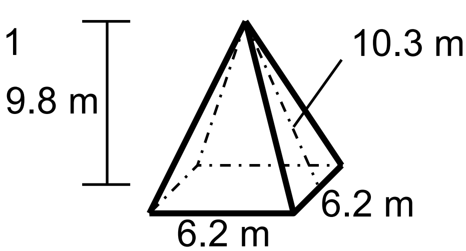
Then

1) Lateral surface area: $L = \frac{1}{2}ps$
2) Total surface area: $SA = L + B$
3) Volume: $V = \frac{1}{3}Bh$

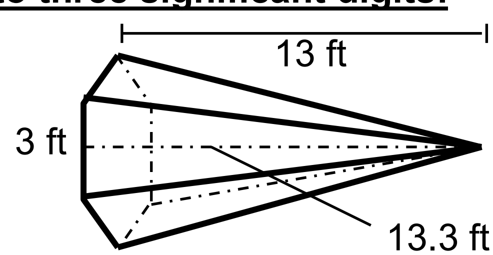


Find a) the lateral surface area, b) the surface area, and c) the volume of the following. Round your answers to three significant digits:

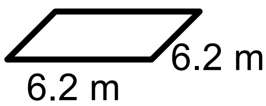
Ex. 1



Ex. 2



Solution:
The base of the pyramid is a square. First, we need to find the perimeter and area of the base:



$p = 4s = 4(6.2) = 24.8$ m

$B = s^2 = (6.2)^2 = 38.44$ m^2

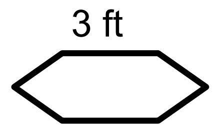
$h = 9.8$ m
$s = 10.3$ m

a) $L = \frac{1}{2}ps = \frac{1}{2}(24.8)(10.3)$
$= 127.72 \approx 128$ m^2

b) $SA = B + L$
$= 38.44 + 127.72 = 166.16$
≈ 166 m^2

c) $V = \frac{1}{3}Bh = \frac{1}{3}(38.44)(9.8)$
$= 125.57\ldots \approx 126$ m^3

Solution:
The base of the pyramid is a regular hexagon. First, we need to find the perimeter and area of the base:



$p = 6s = 6(3) = 18$ ft

$B = \frac{3a^2\sqrt{3}}{2} = \frac{3(3)^2\sqrt{3}}{2} = \frac{27\sqrt{3}}{2}$
$= 23.3826\ldots$ ft^2

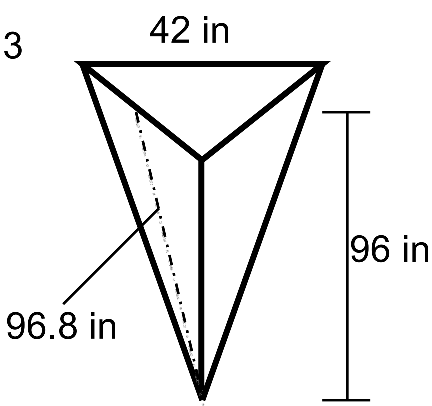
$h = 13$ ft
$s = 13.3$ ft

a) $L = \frac{1}{2}ps = \frac{1}{2}(18)(13.3)$
$= 119.7 \approx 12\bar{0}$ ft^2

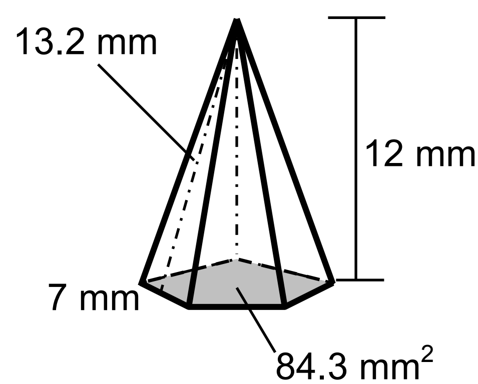
b) $SA = B + L$
$= 23.3826\ldots + 119.7$
$= 143.0826\ldots \approx 143$ ft^2

c) $V = \frac{1}{3}Bh = \frac{1}{3}(23.382\ldots)(13)$
$= 101.324\ldots \approx 101$ ft^3

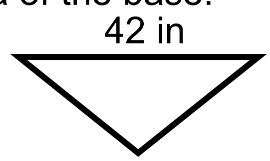
Ex. 3



Ex. 4



Solution:
The base of the pyramid is an equilateral triangle. First, we need to find the perimeter and the area of the base:



p = 42 + 42 + 42 = 126 in

B = $\sqrt{63(63-42)(63-42)(63-42)}$

= $\sqrt{63(21)(21)(21)}$ = $\sqrt{583443}$

= 763.8344… in^2

h = 96 m

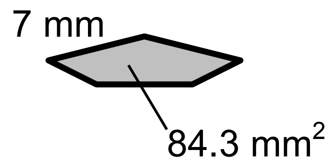
s = 96.8 m

a) L = $\frac{1}{2}$ps = $\frac{1}{2}$(126)(96.8)
= 6098.4 ≈ $61\bar{0}0$ in^2

b) SA = B + L
= 763.8344… + 6098.4
= 6862.23… ≈ 6860 in^2

c) V = $\frac{1}{3}$Bh
= $\frac{1}{3}$(763.8344…)(96)
= 24442.7… ≈ 24,400 in^3

Solution:
The base of the pyramid is a regular pentagon. First, we need to find the perimeter of the base:



p = 7 + 7 + 7 + 7 + 7 = 35 mm

B = 84.3 mm^2

h = 12 mm

s = 13.2 mm

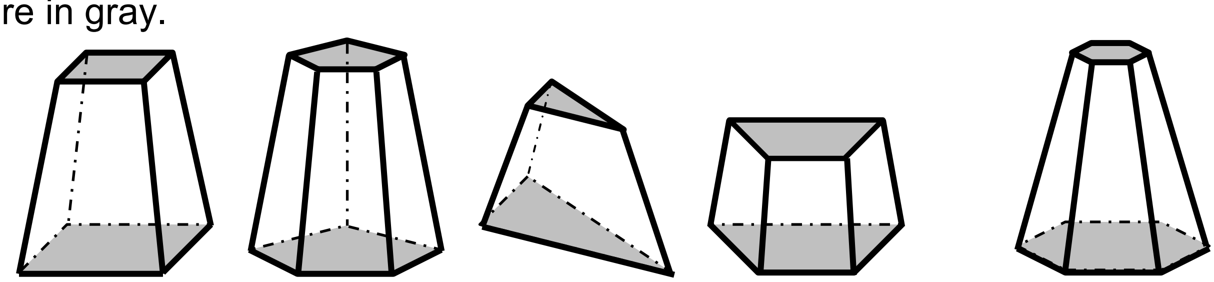
a) L = $\frac{1}{2}$ps = $\frac{1}{2}$(35)(13.2)
= 231 mm^2

b) SA = B + L
= 84.3 + 231
= 315.3 ≈ 315 mm^2

c) V = $\frac{1}{3}$Bh = $\frac{1}{3}$(84.3)(12)
= 337.2 ≈ 337 mm^3

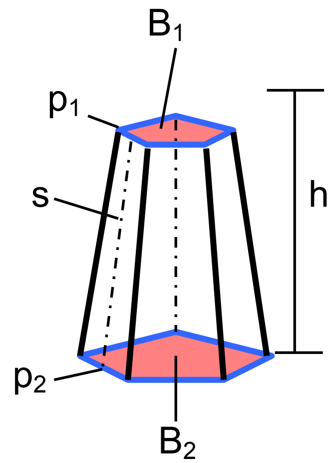
Objective 2: Understanding Frustums, Volumes, and Surface Area.

The **frustum of a pyramid** is the remaining part of the pyramid after cutting off the top portion parallel to the base. Frustum of pyramids have two parallel bases which are similar (the same shape, but are not the same size) and the lateral sides are trapezoids. In each frustum below, the bases are in gray.



Properties of Frustums of Right Pyramids:

Let p_1 = the perimeter of the base on the top
B_1 = the area of the base on the top
p_2 = the perimeter of the base on the bottom
B_2 = the area of the base on the bottom
h = the height of the Frustum
s = the slant height



Then

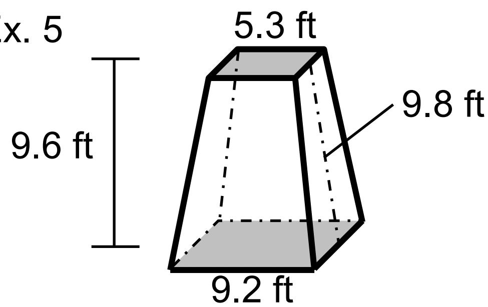
1) Lateral surface area: $L = \frac{1}{2}(p_1 + p_2)s$

2) Total surface area: $SA = L + B_1 + B_2$

3) Volume: $V = \frac{1}{3}(B_1 + B_2 + \sqrt{B_1B_2})h$

Find a) the lateral surface area, b) the surface area, and c) the volume of the following. Round your answers to three significant digits:

Ex. 5



Solution:

$p_1 = 4(5.3) = 21.2$ ft
$B_1 = 5.3(5.3) = 28.09$ ft^2
$h = 9.6$ ft

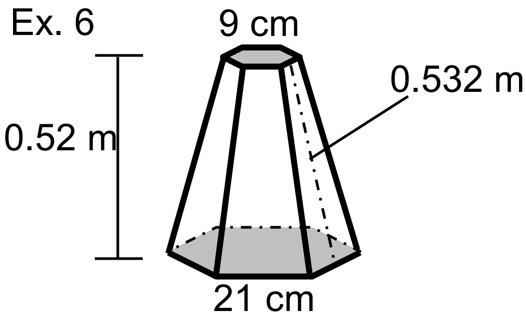
$p_2 = 4(9.2) = 36.8$ ft
$B_2 = 9.2(9.2) = 84.64$ ft^2
$s = 9.8$ ft

a) $L = \frac{1}{2}(p_1 + p_2)s = \frac{1}{2}(21.2 + 36.8)(9.8) = \frac{1}{2}(58)(9.8)$
$= 284.2 \approx 284$ ft^2

b) $SA = L + B_1 + B_2 = 284.2 + 28.09 + 84.64$
$= 396.93 \approx 397$ ft^2

c) $V = \frac{1}{3}(B_1 + B_2 + \sqrt{B_1B_2})h$
$= \frac{1}{3}(28.09 + 84.64 + \sqrt{(28.09)(84.64)})(9.6)$
$= \frac{1}{3}(28.09 + 84.64 + \sqrt{2377.5376})(9.6)$
$= \frac{1}{3}(28.09 + 84.64 + 48.76)(9.6) = \frac{1}{3}(161.49)(9.6)$
$= 516.768 \approx 517$ ft^3

Ex. 6



Solution:

$p_1 = 6(9) = 54$ cm

$B_1 = \frac{3a^2\sqrt{3}}{2} = \frac{3(9)^2\sqrt{3}}{2}$

$= \frac{243\sqrt{3}}{2} = 210.4\ldots$ cm^2

$p_2 = 6(21) = 126$ cm

$B_2 = \frac{3a^2\sqrt{3}}{2} = \frac{3(21)^2\sqrt{3}}{2}$

$= \frac{1323\sqrt{3}}{2} = 1145.7\ldots$ cm^2

We will need to convert the height and the slant height into cm by moving the decimal point two places to the right:

h = 0.52 m = 52 cm s = 0.532 m = 53.2 cm

a) $L = \frac{1}{2}(p_1 + p_2)s = \frac{1}{2}(54 + 126)(53.2) = \frac{1}{2}(180)(53.2)$
$= 4788 \approx 4790$ cm^2

b) $SA = L + B_1 + B_2 = 4788 + 210.4\ldots + 1145.7\ldots$
$= 6144.1\ldots \approx 6140$ cm^2

c) $V = \frac{1}{3}(B_1 + B_2 + \sqrt{B_1B_2})h$
$= \frac{1}{3}(210.4\ldots + 1145.7\ldots + \sqrt{(210.4...)(1145.7...)})(52)$
$= \frac{1}{3}(210.4\ldots + 1145.7\ldots + \sqrt{241116.75})(52)$
$= \frac{1}{3}(210.4\ldots + 1145.7\ldots + 491.03\ldots)(52) = \frac{1}{3}(1847.2\ldots)(52)$
$= 32018.6\ldots \approx 32\bar{0}00$ cm^3