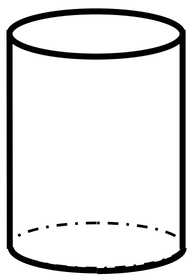
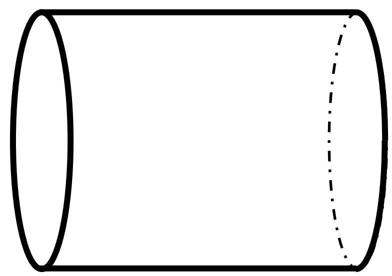
Sect 9.3 – Cylinders and Spheres

Objective 1: Understanding Cylinders, Volume and Surface Area.

A **Cylinder** is a solid figure having congruent circles as bases that are parallel and having sides if folded out flat would form a parallelogram. The perpendicular distance between the bases is called the height or altitude. If the sides of the cylinder are perpendicular to the base, then the cylinder is a right cylinder. We can think of a cylinder as a type of prism. Its base is a circle so the perimeter of the base is $2\pi r$ and the area of the base is πr^2. The lateral surface area is $Ch = 2\pi rh$, and the volume is $Bh = \pi r^2 h$ Thus, the volume for a cylinder is $V = Bh = \pi r^2 h$. Since the two bases are circles, the total surface area is the lateral surface area plus the sum of the areas of the two circles.



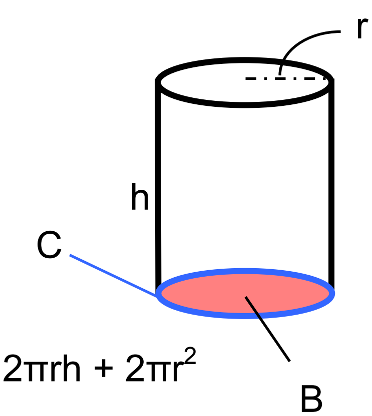


Properties of Right Cylinders:

Let C = the circumference of the base
r = the radius of the base
B = the area of the base
h = the height of the cylinder

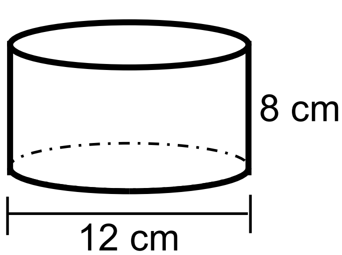
Then

1) Area of the base: $B = \pi r^2$
2) Lateral surface area: $L = Ch = 2\pi rh$
3) Total surface area: $SA = L + 2B = 2\pi rh + 2\pi r^2$
4) Volume: $V = Bh = \pi r^2 h$

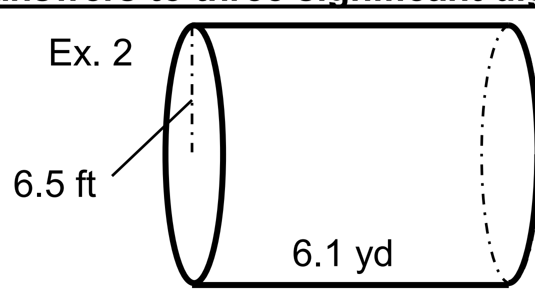


Find a) the lateral surface area, b) the surface area, and c) the volume of the following. Round your answers to three significant digits:

Ex. 1



Ex. 2



Solution:
Since the diameter is 12 cm, then the radius is 12/2 = 6 cm
The height is 8 cm.

$C = 2\pi r = 2\pi(6) = 12\pi$
$= 37.699\ldots$ cm
$B = \pi r^2 = \pi(6)^2 = 36\pi$
$= 113.097\ldots \text{ cm}^2$

a) $L = Ch = (37.699\ldots)(8)$
$= 301.592\ldots \approx 302 \text{ cm}^2$

b) $SA = L + 2B$
$= 301.592\ldots + 2(113.097\ldots)$
$= 301.592\ldots + 226.194\ldots$
$= 527.787\ldots \approx 528 \text{ cm}^2$

c) $V = Bh = (113.097\ldots)(8)$
$= 904.77\ldots \approx 905 \text{ cm}^3$

Solution:
This is a cylinder lying on its side. Thus, radius is 6.5 ft and the height is 6.1 yd = 6.1 yd(3 ft/yd) = 18.3 ft.

$C = 2\pi r = 2\pi(6.5) = 13\pi$
$= 40.84\ldots$ ft
$B = \pi(6.5)^2 = 42.25\pi$
$= 132.73\ldots \text{ ft}^2$

a) $L = Ch = (40.84\ldots)(18.3)$
$= 747.38\ldots \approx 747 \text{ ft}^2$

b) $SA = L + 2B$
$= 747.38\ldots + 2(132.73\ldots)$
$= 747.38\ldots + 265.46\ldots$
$= 1012.84\ldots \approx 1010 \text{ ft}^2$

c) $V = Bh = (132.73\ldots)(18.3)$
$= 2429.000\ldots \approx 2430 \text{ ft}^3$

Ex. 3 How many cubic yards of concrete will need to be poured to create a column with a diameter of 8 ft and a height of 22 ft? If one gallon of red paint can cover 130 ft^2 of the column, how many gallons of paint will be need to cover the top and the sides of the column? Round all answers to the nearest whole number.

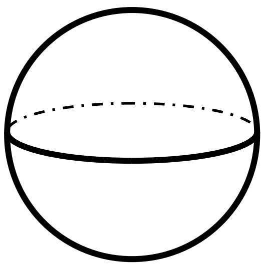
Solution:

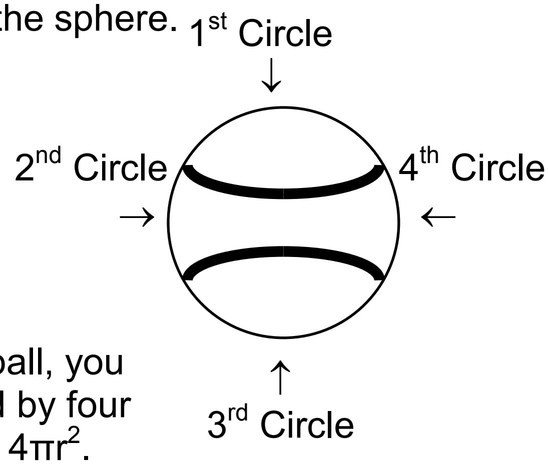
a) The radius is half of the diameter, so r = 8/2 = 4 ft. First, let's calculate the volume:
$V = Bh = \pi r^2 h = \pi(4)^2(22) = \pi(16)(22) = 352\pi \approx 1105.8\ldots \text{ ft}^3$
Now, convert the answer into cubic yards:
$$\frac{1105.8\ldots \text{ ft}^3}{1} \bullet \frac{1\text{yd}^3}{27\text{ft}^3} = 40.95\ldots \approx 41 \text{ yd}^3$$
Forty-one cubic yards of concrete will be needed to pour the column.

b) The lateral surface area is $L = 2\pi rh = 2\pi(4)(22) = 176\pi \approx 552.9\ldots \text{ ft}^2$
The area of the top is $B = \pi r^2 = \pi(4)^2 = 16\pi \approx 50.26\ldots \text{ ft}^2$
The area to be covered then is $L + B = 552.9\ldots + 50.26\ldots = 603.18\ldots \text{ft}^2$
1 gal = 130 ft^2, so 603.18 ft^2 (1 gal/130 ft^2) = 4.63… ≈ 5 gallons.
So, 5 gallons of paint will be needed to cover the column.

Objective 2: Understanding Spheres, Volume, and Surface Area.

A **Sphere** is the set of all points in three dimensions that are the same distance from the center point of the sphere.





If you look at a softball or a baseball, you can see that it is "roughly covered by four circles." Thus, the surface area is $4\pi r^2$.

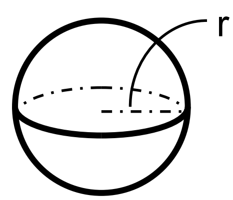
<u>Properties of Spheres:</u>

Let r = the radius of the sphere

Then

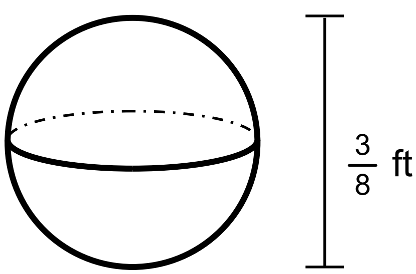
1) Total surface area: $SA = 4\pi r^2$

2) Volume: $V = \frac{4}{3}\pi r^3$



<u>Find a) the surface area, and b) the volume of the following. Round your answers to three significant digits:</u>

Ex. 4



<u>Solution:</u>

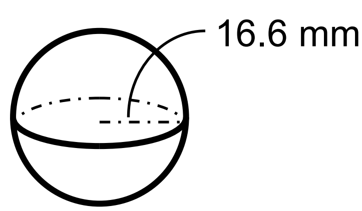
$r = \frac{3}{8} \div 2 = \frac{3}{16}$ ft

a) $SA = 4\pi r^2 = 4\pi\left(\frac{3}{16}\right)^2$

$= 4\pi\frac{9}{256} = \frac{9}{64}\pi$

$= 0.4417\ldots \approx 0.442$ ft^2

Ex. 5



<u>Solution:</u>

r = 16.6 mm

a) $SA = 4\pi r^2 = 4\pi(16.6)^2$

$= 4\pi(275.56) = 1102.24\pi$

$= 3462.789\ldots \approx 3460$ mm^2

b) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{3}{16}\right)^3$
$= \frac{4}{3}\pi(0.0065917\ldots)$
$= 0.0087890625\pi$
$= 0.02761\ldots \approx 0.0276 \text{ ft}^3$

b) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(16.6)^3$
$= \frac{4}{3}\pi(4574.296)$
$= 6099.06\ldots\pi$
$= 19160.7\ldots \approx 19{,}200 \text{ mm}^3$

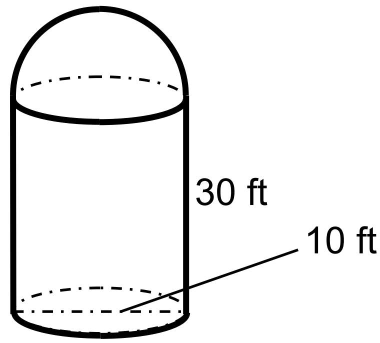
Ex. 6 How many gallons of water can be store in a spherical tank 192 inches in diameter? If water weighs 62.4 lb/ft^3 and the steel used to make the tank weighs 130.6 lb/ft^2, how much does the tank weigh when it is full? Round to four significant digits.

Solution:

a) Since the diameter is 192 inches, the radius is 192/2 = 96 in. Now, plug in to find the volume:
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(96)^3 = \frac{4}{3}\pi(884736) = 1179648\pi$
$= 3705973.4\ldots \text{ in}^3$.
But, 1 gal = 231 in^3, so
$3705973.4\ldots \text{ in}^3 = \frac{3705973.4\ldots \text{in}^3}{1} \bullet \frac{1 \text{gal}}{231 \text{ in}^3} = 16043.17\ldots$
≈ 16040 gal
The tank will hold 16,040 gallons of water.

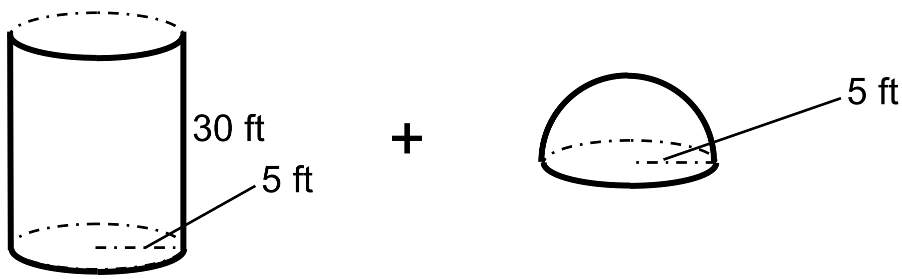
b) First, find the weight of the water. From above, the volume of the tank was 3705973.4… in^3. We will convert this to ft^3.
$3705973.4\ldots \text{ in}^3 = \frac{3705973.4\ldots \text{in}^3}{1} \bullet \frac{1 \text{ ft}^3}{1728 \text{ in}^3} = 2144.6\ldots \text{ ft}^3$
The water weighs 62.4 lb/ft^3. Multiplying, we get:
2144.6… ft^3(62.4 lb/ft^3) = **133826.82… lb**
Now, find the weight of the steel:
$SA = 4\pi r^2 = 4\pi(96)^2 = 4\pi(9216) = 36864\pi = 115811.6\ldots \text{ in}^2$
We will convert this to ft^2:
$115811.6\ldots \text{ in}^2 = \frac{115811.6\ldots \text{in}^2}{1} \bullet \frac{1 \text{ ft}^2}{144 \text{ in}^2} = 804.24\ldots \text{ ft}^2$
The steel weighs 130.6 lb/ft^2. Multiplying, we get:
(804.24… ft^2)(130.6 lb/ft^2) = **105034.7… lb**
Adding the two results together, we get:
133826.82… lb + 105034.7… lb = 238861.57… lb ≈ 238,900 lb
When the tank is full, it will weigh 238,900 lb.

Ex. 7 Farmer Joyce is constructing the silo pictured below. How many bushels of grain will she be able to store in the silo? (round to the nearest hundred bushel).



Solution:

The volume of the figure is equal to the volume of the cylinder plus the volume of the half-sphere or hemisphere. The radius for both the hemisphere and the cylinder is 10 ÷ 2 = 5 ft:



$$V = \pi r^2 h + \frac{4}{3}\pi r^3 \div 2$$

$$= \pi(5)^2(30) + \frac{4}{3}\pi(5)^3 \div 2$$

$$= \pi(25)(30) + \frac{4}{3}\pi(125) \div 2 = 750\pi + (83.3\ldots)\pi$$

$$= 2356.194\ldots + 261.79\ldots = 2617.993\ldots \text{ ft}^3$$

But, 1 bu ≈ 1.24446 ft^3, so

$$2617.993\ldots \text{ ft}^3 \approx \frac{2617.993\ldots \text{ ft}^3}{1} \bullet \frac{1 \text{ bu}}{1.24446 \text{ ft}^3} = 2103.7\ldots \approx 2100 \text{ bu}$$

The silo will hold 2100 bushels.