Coordinate Systems

A coordinate system is used to specify locations in space and consists of

1. A fixed reference point called the origin
2. A set of specified axes or directions with an appropriate scale and label
3. Instructions on how to label a point relative to the origin and axes.

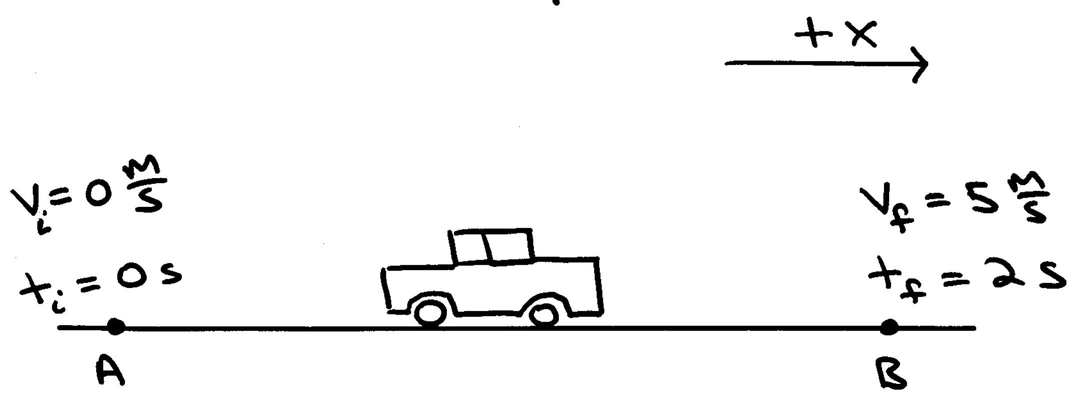
Examples

1. Cartesian (or rectangular) coordinate system - uses x and y.

2. Plane polar coordinate system - uses r and θ.

r is usually taken as the distance from the origin to the point.
θ is taken as the angle between r and the reference line (usually the x-axis in the counterclockwise dir.).

Example: A car starts from rest and moves along a straight line. It reaches a velocity of 5 m/s in 2 s. What is its average acceleration?



Answer

$\bar{a} = 2.5\ m/s^2$

One-Dimensional Motion With Constant Acceleration

Ex. Objects in free fall

When an object moves with constant acceleration

$$a = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

For convenience, let $t_i = 0$ and t_f be any arbitrary time t. Also let $v_i = v_0$ be the object's initial velocity (at $t_i = 0$) and $v_f = v$ be the object's final velocity ($t_f = t$). Substitution gives

$$a = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}$$

$$\therefore \quad \boxed{v = v_0 + at}$$

* Valid only for motion with constant acceleration *

Kinematic Equations for Motion in a straight line under constant acceleration

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$$v = v_0 + at$$

velocity as a function of time

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

Displacement as a function of time

$$v^2 = v_0^2 + 2a(x - x_0)$$

velocity as a function of displacement

$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

Displacement as a function of velocity and time

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* Valid only when the object moves with constant acceleration *

Example: A car uniformly accelerates from 10 m/s to 25 m/s in 5 s. What was its acceleration and how far does it travel in this time?

Answer

$a = 3\ m/s^2$

$x = 87.5\ m$

Example: A jet plane lands with a velocity of 100 m/s and can uniformly accelerate at -5 m/s^2 as it comes to rest.

(a) From the instant it touches the runway, what is the minimum time needed before it stops?

(b) Can this plane land at a small airport where the runway is 0.80 km = 800m long?

Answers

(a) $t = 20$ s

(b) No, needs 1000 m to land safely.

Problem Solving Strategy

For motion in a straight line under constant acceleration

1. Draw a simple, neat diagram of the system

2. Choose a coordinate system and define the directions

3. Make a list of all known and unknown quantities.

4. Select from the list of kinematic equations the one (or ones) that will allow the unknowns to be determined.

5. If time permits, double check your results.

Example: A speedboat increases in speed from 20 m/s to 30 m/s in a distance of 200 m. Find

(a) the magnitude of the acceleration.
(b) the time it takes the boat to travel this distance.

Answers

(a) $a = 1.25\ m/s^2$
(b) $t = 8\ s$