MGMG 522 : Session #3
Hypothesis Testing

(Ch. 5)

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What Is Hypothesis Testing?

- It is usually the step you normally do after obtaining the estimates.
- Hypothesis testing does not <u>prove or disprove</u> the theory. It merely shows whether or not there is some evidence from the sample data to show some support for the theory.
- Hypothesis testing tells us how likely the result that we get is obtained purely by chance.
- Using hypothesis testing, we can reject a certain hypothesis with a <u>certain level of confidence</u>, not with complete confidence.

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Specifying Null and Alternative Hypotheses

- You should specify null and alternative hypotheses before you see the results. Had you seen the results first, you might have been tempted to change your null and alternative hypotheses.
- The hypotheses should be based on theory.
- Null hypothesis is “what you expect to occur if your theory is not correct.”
- Alternative hypothesis is “what you expect to occur if your theory is correct.”

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Null and Alternative Hypotheses

- We can control the probability of rejecting the null hypothesis when it is true.
- But we never know the probability of accepting the null hypothesis as being correct when it is, in fact, false.
- Therefore, we will never ever say we accept the null hypothesis. We would rather say, we fail to (or cannot) reject the null hypothesis. It is like the judge will always say the defendant is “not guilty.” [instead of “innocent”]
- Sometimes, some researchers will say they “accept” the null hypothesis, but the word “accept” is put in quotes.

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Example 1 of Hypotheses

- If you think X_1 is one explanatory factor in your regression model, then the coefficient in front of X_1 should not be equal to zero.
- Your null and alternative hypotheses will be as follow:

H_0: $\beta_1 = 0$
H_1: $\beta_1 \neq 0$ (your theory: X_1 = explanatory factor)

- This is called a two-sided (or two-tailed) test.

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Example 2 of Hypotheses

- If you think X_1 is positively related to Y in your regression model, then the coefficient in front of X_1 should be greater than zero.
- Your null and alternative hypotheses will be as follow:

H_0: $\beta_1 \leq 0$

H_1: $\beta_1 > 0$ (your theory: X_1 is positively related to Y)

- This is called a one-sided (or one-tailed) test.

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Example 3 of Hypotheses

- If you think X_1 is negatively related to Y in your regression model, then the coefficient in front of X_1 should be less than zero.
- Your null and alternative hypotheses will be as follow:

  H_0: $\beta_1 \geq 0$
  H_1: $\beta_1 < 0$ (your theory: X_1 is negatively related to Y)

- This is also called a one-sided (or one-tailed) test.
- Q: When to conduct a one-sided or a two-sided t-test?

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Type I and Type II Errors

- Type I: Reject the true null hypothesis
- Type II: Do not reject a false null hypothesis
- Example: In a murder trial, the court system usually follows the presumption of "innocent until proven guilty beyond reasonable doubts."
- That translates into

  H_0: The defendant is innocent.
  H_1: The defendant is guilty.

- Type I error implies "sending an innocent to jail."
- Type II error implies "letting a murderer go free."
- Which is more serious? We have to weigh the costs.
- There is one problem though, that is, reducing the chance of making Type I error will increase the chance of making Type II error.
- In the example above, if we are afraid too much of sending innocents to jail, and hence, never send anyone to jail, we will probably free a lot of murderers!

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Decision Rules

- We need to establish a decision rule before we can decide whether or not to reject or not reject a null hypothesis.
- This rule should be established before we obtain the estimates to reduce bias.
- The decision rule provides us with a border line that separates the acceptance region from the rejection region.
- The area (probability) in the rejection region is the probability of making Type I error.

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t-Test

- “t” as in “t-test” is always spelled with a lowercase letter.
- We use t-test to test one coefficient estimate at a time.
- Never use a t-test to test the intercept term!
- To test more than one coefficient estimate at a time, we use F-test. [F is always capitalized.]
- t-test is easy to use and robust.
- t-test is the appropriate test to use in a regression analysis with normally distributed error term and when the variance of the error term needs to be estimated.

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t-Statistic

- t-statistic for k^{th} coefficient is, $t_k = \frac{\hat{\beta}_k - \beta_{H_0}}{SE(\hat{\beta}_k)}$

  $\hat{\beta}_k$ = coefficient estimate

  β_{H_0} = hypothesized value of the coefficient

  $SE(\hat{\beta}_k)$ = estimated standard error of $\hat{\beta}_k$

- t-statistic follows a t-distribution.
- t-distribution is a bell-shaped symmetrical distribution.
- Try to avoid assigning a number other than zero to β_{H_0}. Why?

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t-value, Critical t-value, Degrees of Freedom, and Level of Significance

- t-value is calculated from the t-statistic formula.
- Critical t-value is the t-value that separates the acceptance region from the rejection region.
- Critical t-value for one-sided t-test at X% level of significance is always the same critical t-value for two-sided t-test at 2X% level of significance.
- Level of significance specifies the probability of Type I error.
- Degrees of freedom for t-test of coefficient estimate = n - K – 1.

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Two Common Approaches in Hypothesis Testing

1. Approach 1: Specify the level of significance before doing hypothesis testing

   Example: Choose 0.05 level of significance.

   - If p-value > 0.05, fail to reject H_0.
   - If p-value < 0.05, reject H_0.
   - What if p-value = 0.05?

   Q: Should we select a very low level of significance?

2. Approach 2: Calculate the p-value first and choose the level of significance later

   Example: Look up for the p-value associated with a particular t-value with n-k-1 degrees of freedom from the t-distribution table, then, decide whether or not to reject H_0 at what level of significance.

   Caution: You might be tempted to vary the level of significance so that you can reject H_0.

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What if the signs of the coefficients are unexpected and/or insignificant?

- It doesn't mean that those independent variables are not important or are not related to the Y variable in your regression model. This could happen by chance.
- If the sign is unexpected and significant, do not change your null and alternative hypotheses so that the result agrees with your data. If you do so, you are changing your old theory and accepting a new theory based solely on this sample.
- What you would rather do in this case is, to conclude that you do not find any support for your hypothesis (or theory) from this sample data.

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Other Use of t-Test

- t-test is not for testing a coefficient estimate only.
- We can use t-test to test the simple correlation coefficient, r.
- The simple correlation coefficient, r, measures the direction and magnitude of the linear relationship between two variables.
- $-1 \leq r \leq 1$
- Example of use: Test for imperfect multicollinearity problem between X_i and X_j, for $i \neq j$
- $t = \frac{r\sqrt{n-2}}{\sqrt{(1-r^2)}}$, degrees of freedom = n - 2

Limitations of t-Test

1. t-test does not prove the theory.
2. t-test does not tell which coefficient is more important or which factor has more effect on Y.
3. t-test is appropriate for a sample that is small relative to the population. If the size of the sample is large relative to the size of the population, we can almost always reject the null hypothesis. Why?

F-Test

- F-test simultaneously tests whether all coefficients are equal to zero.

  Why F-test does not test whether the intercept term is equal to zero at the same time too?
- H_0: $\beta_1 = \beta_2 = \beta_3 = \ldots = \beta_K = 0$
- H_1: H_0 is not true.

$$F = \frac{ESS/K}{RSS/(n-K-1)}$$ with (K, n-K-1) degrees of freedom

- If we reject H_0, it means that the regression equation's fit is better than that of a naïve model, $\bar{Y}$.

F-Test (continue…)

- F-value is a function of the R^2. It can be shown that

$$F = \frac{R^2/K}{(1-R^2)/(n-K-1)}$$

- F is a statistical measure of fit, while R^2 is just a qualitative measure. F-test can tell how good the overall fit is, while R^2 cannot.
- For a simple regression, $F = t^2$. F-test and t-test are identical. They will lead to the same "accept/reject H_0" decision. [This is not true for a multiple regression.]
- F-test jointly tests the coefficients, while t-test tests the coefficient individually.
  - If F-test rejects H_0, t-test on individual coefficient can reject or "accept" H_0.
  - If F-test "accepts" H_0, t-test on individual coefficient can also reject or "accept" H_0.

Normality Tests

- Look at the histogram of a variable to see if it has a bell shape.
- Use the Anderson-Darling normality test (A^2 statistic)
  - The null hypothesis, H0: the variable in question is normally distributed.
- Use the Jarque-Bera test of normality
  - The null hypothesis, H0: the variable in question is normally distributed.
- To conduct the normality test of the residuals
  - Estimate an equation and then click on "View/Residual Tests/Histogram - Normality Test"

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