GEO 622004 reading (3pgs) 1 of 3

Lesson 2

Conditional Statements

These three ads have something in common: each has a headline that begins with the word "if." Statements consisting of two clauses, one of which begins with the word "if" or "when" or some equivalent word, are called *conditional statements*. Such statements are often used when the purpose is to establish certain conclusions and so they are very common in the field of advertising. They are also important in mathematics in writing deductive proofs.

COURTESY OF AVIS RENT A CAR SYSTEM, INC.

COURTESY OF COLUMBIA BROADCASTING SYSTEM

A conditional statement can be represented symbolically by "If a, then b." or, even more briefly, by

$$a \rightarrow b.$$

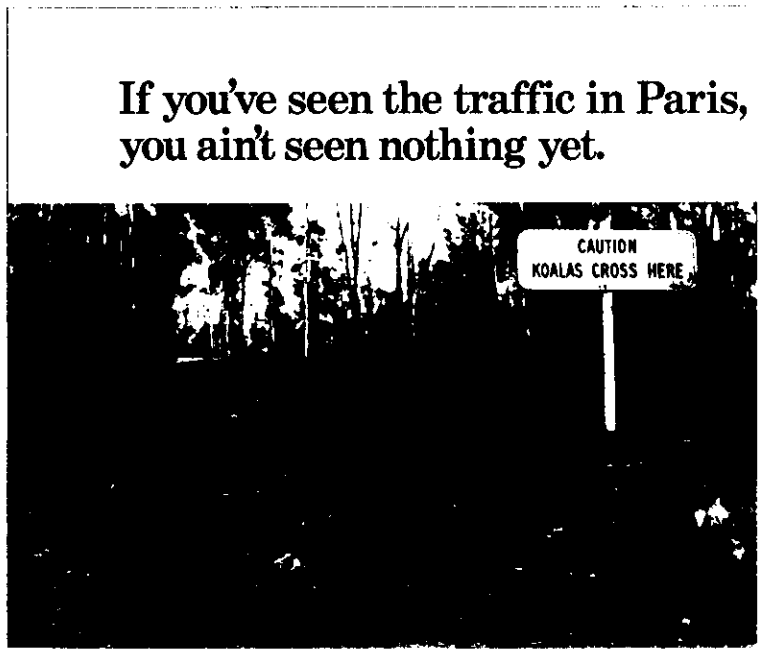
The letter a represents the "if" clause, or *hypothesis*, and the letter b represents the "then" clause, or *conclusion*. (The word "then," being understood, is usually omitted.) The symbols

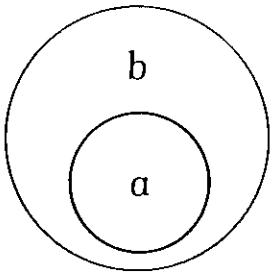
$$a \rightarrow b$$

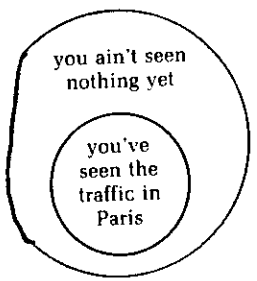
are read as "if a, then b." or as "a implies b." For example, in the first ad, a represents the words "Avis is out of cars" and b represents the words "we'll get you one from our competition." In the second ad, a stands for "life is discovered on Mars" and b for "it will come as news to you," and so on.

It is helpful in learning how to relate two or more conditional statements to each other to be able to represent them with circle diagrams. These are often called *Euler diagrams* after an eighteenth-century Swiss mathematician, Leonhard Euler, who first used them.

COURTESY OF QANTAS - AUSTRALIA'S ROUND-THE-WORLD AIRLINE







To represent a conditional statement with an Euler diagram, we draw two circles, one inside the other. The interior of the smaller circle represents a, the hypothesis, and the interior of the larger circle represents b, the conclusion. Notice that if a point is inside circle a, it is also inside circle b. Or, more briefly, "if a, then b." which is what the diagram is intended to represent. The headline of the last ad, represented by an Euler diagram, looks like the figure shown at the left.

Exercises

Beginning with this lesson, answers to many of the exercises are provided at the back of this book so that you can check some of your work as soon as you have finished it.

SET I

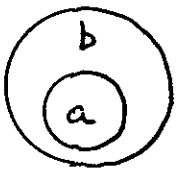
Conditional statements are not always written in the form "if a, then b." Rewrite each of the following sentences in "if-then" form. Be careful not to change the meanings of any of the sentences; for instance, if you write a true conditional statement in "if-then" form so that it turns out to be false, something is wrong.

Example. A baby sneezes when it gets pepper in its nose.
Answer. If a baby gets pepper in its nose, then it sneezes.

1. When you cross your eyes, I crack up.
2. Smokey the Bear wouldn't have to do commercials for a living if money grew on trees.
3. All surfers like big waves.
4. Licorice-flavored ice cream has a peculiar color.
5. A heavy object stored in the attic of a jungle mansion may crash down upon the occupants.
6. People who live in grass houses shouldn't stow thrones.
7. No ghost has a shadow.

SET II

1. Draw an Euler diagram to represent the following statement:
   All pelicans eat fish.
2. Several "if-then" statements are listed below. Which of them seem to be true if the diagram you have drawn represents a true statement?
   a) If a bird is a pelican, then it eats fish.
   b) If a creature eats fish, then it is a pelican.
   c) If a bird is not a pelican, then it doesn't eat fish.
   d) If a creature doesn't eat fish, then it is not a pelican.
3. Draw an Euler diagram to represent the statement:
   Pro basketball players are not midgets.
4. Which of the statements below are true if your diagram represents a true statement?
   a) If a fellow is a pro basketball player, then he is not a midget.
   b) If a fellow is not a pro basketball player, then he is a midget.
   c) If a fellow is not a midget, then he is a pro basketball player.
   d) If a fellow is a midget, then he is not a pro basketball player.
5. You know that the general conditional statement $a \rightarrow b$ is represented by the diagram shown here. Which of the following "if-then" statements does it represent?
   a) If b, then a.
   b) If not a, then not b.
   c) If a, then b.
   d) If not b, then not a.



6. How many "if-then" statements does an Euler diagram represent?

SET III

B.C. has made an analogy in this cartoon between flying and traveling by wheel. Peter said "If God had wanted you to fly, he would have given you wings."

1. What "if-then" statement is implied by B.C.'s analogy?

2. Can you write a different "if-then" statement that has the same meaning?

Lesson 3
Equivalent Statements

Lewis Carroll, the author of *Alice's Adventures in Wonderland* and *Through the Looking Glass*, was a mathematics teacher who wrote stories as a hobby. His books contain many amusing examples of both good and deliberately poor logic and, as a result, have long been favorites among mathematicians. Consider the following conversation held at the Mad Hatter's Tea Party.

"Then you should say what you mean," the March Hare went on.

"I do," Alice hastily replied; "at least—at least I mean what I say—that's the same thing, you know."

"Not the same thing a bit!" said the Hatter. "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"

"You might just as well say," added the March Hare, "that 'I like what I get' is the same thing as 'I get what I like'!"

"You might just as well say," added the Dormouse, who seemed to be talking in his sleep, "that 'I breathe when I sleep' is the same thing as 'I sleep when I breathe'!"

"It *is* the same thing with you," said the Hatter, and here the conversation dropped, and the party sat silent for a minute.