

Cooperative Learning Activity

For use with pages 96-101

GOAL

To understand the use of rules, properties and logic, similar to algebraic properties, to prove theorems and solve problems

Exploring Algebraic Reasoning

Many algebraic properties, such as the addition, subtraction, and multiplication properties, make it possible to solve problems. Understanding the rules of mathematics is imperative to the learning of these properties. Being able to follow rules in a logical and organized fashion is necessary to fully grasp algebraic concepts. In this activity you will be given a set of rules, similar to properties of algebra, and you will be asked to use these rules and logic to prove a given theorem.

Instructions

Three rules govern the usage of the numbers 1, 2, 3, and 4. A "theorem" is a string of numbers followed by \gg followed by a string. By using the three rules, it is possible to change one string of numbers into another string. For example, 1, 2, 2, 3, 4, 4 \gg 2, 3, 2 says that by using the rules, 1, 2, 2, 3, 4, 4 can be transformed into 2, 3, 2. Your group will have to prove or disprove a number of challenge strings. The three rules are:

- ① If a string ends in the same two numbers, then you may substitute a "2" in place of those two numbers. For example, 3, 4, 4 \gg 3, 2. This is similar to the substitution property.
- ② If a string of numbers starts and finishes with the same number, then you may substitute a "3" in place of all the numbers between the first and last numbers. For example 1, 2, 3, 4, 1 \gg 1, 3, 1. This is similar to the substitution property.
- ③ Any two adjacent numbers can change place with each other. For example, 3, 4, \gg 4, 3. This is similar to the transitive property.

Example

Show that 1, 2, 2, 3, 4, 4 \gg 2, 3, 2

Proof	1, 2, 2, 3, 4, 4	Given
	1, 2, 2, 3, 2	Rule 1
	2, 1, 2, 3, 2	Rule 3
	2, 3, 2	Rule 2

So 1, 2, 2, 3, 4, 4 \gg 2, 3, 2.

As a group, use the rules to prove the theorems below.

- | | |
|--------------------------------------------------|--------------------------------|
| 1. 1, 2, 1, 3, 2 \gg 3, 2 | 2. 1, 2, 2, 2, 2, 1 \gg 3, 2 |
| 3. 1, 2, 3, 4, 4, 2, 3, \gg 3, 2 | 4. 1, 4, 4, 3, 4, \gg 3, 2 |
| 5. 1, 4, 3, 2, 2, 3, 4, 2, 1, 4, 4, 4 \gg 3, 2 | |

Real-Life Application: When Will I Ever Use This?

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Temperature Scales

The most common temperature scales are Fahrenheit and Celsius. The United States is one of the few countries in the world that does not predominantly use the Celsius scale for everyday temperature measurement. In some cases, both types of measurement are used. Weather reports, for example, usually give a temperature reading in both Celsius and Fahrenheit. Nevertheless, in the United States, the Fahrenheit scale is more commonly used than the Celsius scale.

The Swedish astronomer Anders Celsius developed the Celsius scale in the mid-1700s while the Fahrenheit scale was developed around the same time by a German physicist, Gabriel Daniel Fahrenheit. On a Celsius scale, 0° is the freezing point of water and 100° is its boiling point. This is different on a Fahrenheit scale, where the freezing point of water is 32° and its boiling point is 212° .

In Exercises 1–4, use the following information.

It is sometimes necessary to convert degrees Celsius to degrees Fahrenheit or vice versa. To convert from Celsius to Fahrenheit, you can use the formula

$$F = \frac{9}{5}C + 32$$

where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

- Convert each of the following measurements to degrees Fahrenheit.
 - 20°C
 - 30°C
 - -5°C
- Solve the conversion formula for C and write a reason for each step.
- Write another equation that is equal to your answer in Exercise 2. What property should you have used to obtain your equation?
- Convert each of the following measurements to degrees Celsius.
 - 95°F
 - 14°F
 - -17.5°F