Qatar University
The Foundation Program

Semester Fall 2005
Math & Computer Dept.

Math 3 — **Worksheet#2** — **Dec 7, 2005**

1. Solve for x.

a. $\sqrt{2x+6}-\sqrt{x+4}=1$ b. $\frac{1}{x+2}+\frac{1}{x-2}=1$ c. $\frac{7}{x-1}-\frac{2}{\sqrt{x-1}}+\frac{1}{7}=0$

2. Solve for y in terms of x.

a. $(y+2x)^2-8(y+2x)+15=0$ b. $\frac{1}{y+2}=\frac{x-3}{1-x}-1$ c. $\sqrt[3]{\frac{2y+x}{y-x}}=-2$

3. Find all real solutions if any of each equation:

a. $2\sqrt[3]{x^2}-1=\sqrt[3]{x}$ b. $6x^{-1}-5x^{-\frac{1}{2}}+1=0$ c. $\sqrt[3]{(x-1)^2+4}=2$

4. Solve and express your answer in interval notation.

a. $\frac{1-x}{3}-4>\frac{1+2x}{5}$ b. $\frac{r}{3}-\frac{2-3r}{4}\le\frac{1}{5}-\frac{1+r}{6}$ c. $-2(x+3)+3<1+\frac{x}{2}$

5. Solve the following inequalities.

a. $\frac{x+2}{-2x-1}>0$ b. $\frac{2-x}{3x-2}\le 1$ c. $\frac{2}{2+x}\ge\frac{-1}{2-x}$ d. $\frac{4}{x}+\frac{3}{x-2}<1$

6. Solve and express your answer in interval notation.

a. $-2\le\frac{1}{3}(4-3x)<1$ b. $1<\frac{1-2x}{x-2}\le 3$ c. $\frac{1-x}{2+x}\le 2<\frac{1+2x}{x+2}$

7. Solve the following (Express your answer in interval notation)

a. $\sqrt{(1-3x)^2}<1$ b. $\left|\frac{1-x}{x+3}\right|>2$ c. $-1\le\left|\frac{1-3x}{-2}\right|<4$ d. $\frac{1}{|4-x|}<\frac{1}{|x+7|}$

8. Solve the following inequalities.

a. $x^2-3x+1>3$ b. $x^2+x+4\le 1$ c. $1-x^2>3x+4$

9. Solve the following (Express your answer in interval notation)

a. $(x+1)(x^2-x+2)\le 0$ b. $2x^3-4x^2>30x$ c. $x^4>5-4x^2$ d. $16x^4-81<0$

10. Solve the following :

a. $x^2-2x\le 3$ b. $1-3x^2>4$ c. $1<x^2-2x\le 3$ d. $-1<|2x^2+3x|\le 2$

11. Solve the following equations

a. $\left|\frac{1-x}{x+3}\right|=2$ b. $|1-2x|+|3+x|=3$ c. $|x+2|+|1-x|=2x-1$

c. $|2-3x|=|3+2x|$

(1) (a) $\sqrt{2x+6} - \sqrt{x+4} = 1 \Rightarrow \sqrt{2x+6} = 1 + \sqrt{x+4}$

square both sides $\Rightarrow (2x+6) = 1 + (x+4) + 2(1)\sqrt{x+4}$

$\Rightarrow 2\sqrt{x+4} = 2x+6-1-x-4 \Rightarrow 2\sqrt{x+4} = x+1$

square both sides $\Rightarrow 4(x+4) = (x+1)^2$

$\Rightarrow 4x+16 = x^2+2x+1 \Rightarrow x^2+2x+1-4x-16=0$

$\Rightarrow x^2-2x-15=0 \quad x = \frac{2 \pm \sqrt{4-4(1)(-15)}}{2} = \frac{2 \pm \sqrt{64}}{2}$

$\Rightarrow x = \frac{2 \pm 8}{2} \Rightarrow \boxed{x = 5, -3}$

Checking $\boxed{x=5}$ $\sqrt{10+6} - \sqrt{9} = 1$

$4 - 3 = 1$

$1 = 1$

$\boxed{x=-3}$ $\sqrt{-6+6} - \sqrt{-3+4} = 1$

$-1 = 1 \quad \times$

$\Rightarrow S.S. = \{5\}$

(b) L.C.D

(c) $\frac{7}{x-1} - \frac{2}{\sqrt{x-1}} + \frac{1}{7} = 0$ $\quad LCD = 7(x-1)\sqrt{x-1}$

$\Rightarrow 7 \cdot 7 \cdot \sqrt{x-1} - 2 \cdot 7 \cdot (x-1) + (x-1)\sqrt{x-1} = 0$

· let $\sqrt{x-1} = t \Rightarrow (x-1) = t^2$

$49t - 14t^2 + tt^2 = 0$

$t(49 - 14t + t^2) = 0$

$\Rightarrow t = 0$ or $t^2 - 14t + 49 = 0$

$t = 0$ or $(t-7)^2 = 0 \Rightarrow t - 7 = 0$

$t = 7$

$\sqrt{x-1} = 0 \Rightarrow \boxed{x = 1}$

$\sqrt{x-1} = 7 \Rightarrow x - 1 = 49 \Rightarrow \boxed{x = 50}$

Checking $\boxed{x=1}$ make denominator $= 0$ $\quad x = 1$ (×)

(✓) $\boxed{x=50} \Rightarrow \frac{7}{49} - \frac{2}{7} + \frac{1}{7} = 0$

$\frac{1}{7} - \frac{2}{7} + \frac{1}{7} = 0 \Rightarrow 0 = 0$

(2) (a) $(y+2x)^2 - 8(y+2x) + 15 = 0$

$$y+2x = \frac{8 \pm \sqrt{64 - 4\cdot 1\cdot 15}}{2} = \frac{8 \pm \sqrt{4}}{2} = \frac{8 \pm 2}{2} = 5, 3$$

$y + 2x = 5$ OR $y + 2x = 3$

$y = 5 - 2x$ OR $y = 3 - 2x$

(b) LCD $(y+2)(1-x) \Rightarrow (1-x) = (x-3)(y+2) - (y+2)(1-x)$

$\Rightarrow 1 - x = xy + 2x - 3y - 6 - y + xy - 2 + 2x$

$1 - x - 2x + 6 + 2 - 2x = xy - 3y - y + xy$

$9 - 5x = y(x - 3 - 1 + x)$

$9 - 5x = y(2x - 4)$

$$y = \frac{(9 - 5x)}{2(x-2)}$$

(c)

$$\left(\frac{2y+x}{y-x}\right) = (-2)^3 \Rightarrow \frac{2y+x}{y-x} = -8 \Rightarrow 2y + x = -8y + 8x$$

$\Rightarrow 2y + 8y = 8x - x \Rightarrow 10y = 7x \Rightarrow \boxed{y = \frac{7}{10}x}$

(3) (a) $2x^{2/3} - x^{1/3} - 1 = 0$ let $t = x^{1/3} \Rightarrow 2t^2 - t - 1 = 0$

$$t = \frac{1 \pm \sqrt{1 - 4(2)(-1)}}{2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} = 2, -1$$

$x^{1/3} = 2 \Rightarrow \boxed{x = 8}$, $x^{1/3} = -1 \Rightarrow \boxed{x = -1}$

Checking $\boxed{x = 8}$ $2(8)^{2/3} - (8)^{1/3} - 1 = 0$?

$\Rightarrow 2(2^3)^{2/3} - (2^3)^{1/3} - 1 = 0$? $\Rightarrow 2 \cdot 2^2 - 2 - 1 = 0$?

$8 - 2 - 1 = 0$? ✓ $\boxed{x = -1}$ $2(-1)^{2/3} - (-1)^{1/3} - 1 = 0$?

$2 + 1 - 1 = 0$?

$2 = 0$? ✗

no solution

(b) let $\frac{6}{x} + \frac{5}{\sqrt{x}} + 1 = 0 \Rightarrow$ LCD $= x\sqrt{x}$

$\Rightarrow 6\sqrt{x} + 5x + x\sqrt{x} = 0$ let $\sqrt{x} = t$, $x = t^2$

$6t + 5t^2 + t^2 t = 0 \Rightarrow t(6 + 5t + t^2) = 0$

$t = 0$, $t = -3, -2$ | $x = t^2 \Rightarrow x = 0, 9, 4$

Do checking

Q.3 (c) $(x-1)^2 + 4 = 8 \Rightarrow (x-1)^2 = 4$

$x - 1 = \pm 2 + 1 \Rightarrow x = \pm 2 + 1$

$x = 3, -1$ Do checking yourself

(4) (a) $LCD=15 \Rightarrow 5(1-x)-4\cdot 15 > (1+2x)3 \Rightarrow 5-5x-60>3+6x$

$-5x-6x > 3+60-5 \Rightarrow -11x > 58 \Rightarrow x < -58/5$, $(-\infty, \frac{-58}{5})$

(b) $LCD = 60 \Rightarrow 20x - 15(2-3x) \le 12 - 10(1+x)$

$\Rightarrow 20x - 30 + 45x \le 12 - 10 - 10x \Rightarrow 75x \le 32$

$\Rightarrow x \le \frac{32}{75}$ $\quad (-\infty, 32/75]$

(c) $LCD = 2 \Rightarrow -4(x+3)+6 < 2 + x \Rightarrow -4x-12+6<2+x$

$\Rightarrow -4x-x < 2-6+12 \Rightarrow -5x < 8 \Rightarrow x > -8/5$

$(-8/5, \infty)$

(5) (a) $\frac{x+2}{-2x-1} > 0 \Rightarrow \frac{x+2}{-(2x+1)} > 0 \Rightarrow \frac{x+2}{2x+1} < 0$

	-3		-2		0		$-1/2$		1
$x+2$	$-$		0		$+$		$+$		$+$
$2x+1$	$-$		$-$		$+$		0		$+$
	$+$		0		$-$		∞		$+$

$\Rightarrow (x<-2) \cup (x > -\frac{1}{2})$

$\mathbb{R} - [-2, -1/2]$

(b) $\frac{2-x}{3x-2} \le 1 \Rightarrow \frac{2-x}{3x-2} - 1 \le 0 \Rightarrow \frac{(2-x)-(3x-2)}{(3x-2)} \le 0$

$\frac{2-x-3x+2}{3x-2} \le 0 \Rightarrow \frac{-4x+4}{3x-2} \le 0$

$-4x+4=0 \Rightarrow x=1$, $3x-2=0 \Rightarrow x=2/3$

	0		$2/3$		1	
$-4x+4$	$+$		$+$		0	$-$
$3x-2$	$-$		0		$+$	$+$
	$-$		∞		0	$-$

$(-\infty, 2/3) \cup [1, \infty)$

$(x < 2/3) \cup (x \ge 1)$

$\mathbb{R} - [2/3, 1)$

(c) $\frac{2}{2+x} \ge \frac{-1}{2-x} \Rightarrow \frac{2}{2+x} + \frac{1}{2-x} \ge 0 \Rightarrow \frac{2(2-x)+1(2+x)}{(2+x)(2-x)} \ge 0$

$\Rightarrow \frac{4-2x+2+x}{(2+x)(2-x)} \ge 0 \Rightarrow \frac{-x+6}{(2+x)(2-x)} \ge 0$

		-2		2		6	
$-x+6$	$+$	$+$	$+$	$+$	$+$	0	$-$
$2+x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$	$+$	$+$	$+$	0	$-$	$-$	$-$
	$-$	∞	$+$	∞	$-$	0	$+$

$(-2, 2) \cup [6, \infty)$

(6) (a) $-2 \le \frac{1}{3}(4-3x) < 1 \Rightarrow -6 \le (4-3x) < 3$

$\Rightarrow -6-4 \le -3x \le 3-4 \Rightarrow -10 \le -3x < -1$

$\Rightarrow \frac{-10}{-3} \ge x > \frac{-1}{-3} \Rightarrow \boxed{\frac{1}{3} < x \le \frac{10}{3}}$ $\boxed{(\frac{1}{3}, \frac{10}{3}]}$

(b) $1 < \frac{1-2x}{x-2} \le 3 \Rightarrow 1 < \frac{1-2x}{x-2} \cap \frac{1-2x}{x-2} \le 3$

$1 - (\frac{1-2x}{x-2}) < 0 \cap \frac{1-2x}{x-2} - 3 \le 0$

$\frac{(x-2)-(1-2x)}{(x-2)} < 0 \cap \frac{(1-2x)-3(x-2)}{x-2} \le 0$

$\frac{x-2-1+2x}{x-2} < 0 \cap \frac{[(1-2x)-3x+6]}{(x-2)} \le 0$

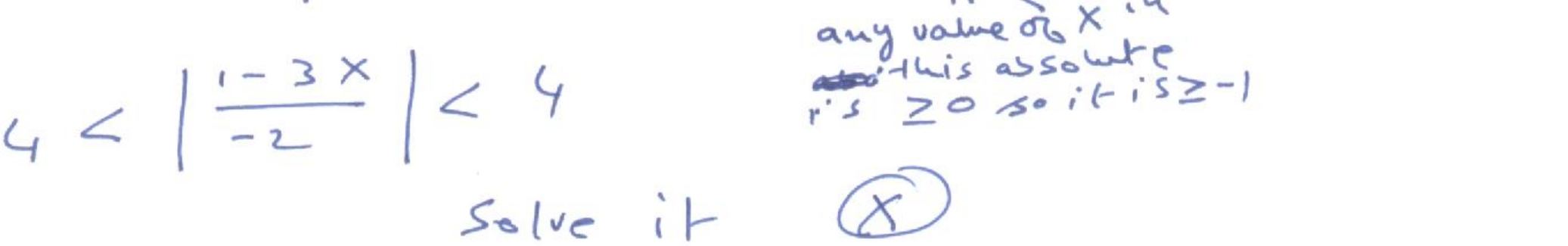
$\frac{3x-3}{x-2} < 0 \cap \frac{7-5x}{x-2} \le 0$

		1		2	
$3x-3$	−	0	+	+	+
$x-2$	−	−	−	0	+
	+	0	−	∞	+

$(1,2)$

		7/5		2	
$7-5x$	+	0	−	−	−
$x-2$	−	−	−	0	+
	−	0	+	∞	−

$(-\infty, 7/5) \cup (2, \infty)$



$\Rightarrow (1, 7/5)$

(c) DO IT YOURSELF

(7) (a) $|1-3x| < 1 \Rightarrow -1 < 1-3x < 1$ solve it

(b) $\frac{1-x}{x+3} < -2 \cup \frac{1-x}{x+3} > +2$ solve it

(c) $-1 \le \left|\frac{1-3x}{-2}\right| < 4 \Rightarrow -1 \le \left|\frac{1-3x}{-2}\right| \cap \left|\frac{1-3x}{-2}\right| < 4$

$\mathbb{R}$ any value of x in this absolute is ≥ 0 so it is ≥ -1

$-4 < \left|\frac{1-3x}{-2}\right| < 4$

Solve it (X)

(8) (a) $x^2-3x+1 > 3 \Rightarrow x^2-3x-2>0 \Rightarrow (x-2)(x-1)>0$

		1		2	
$x-2$	−	−	−	0	+
$x-1$	−	0	+	+	+
	+	0	−	0	+

$(x<1) \cup (x>2)$

$(-\infty, 1) \cup (2, \infty)$

$\mathbb{R} - [1,2]$

(b) $x^2+x+3 \leq 0$

$x = \frac{-1 \pm \sqrt{1-12}}{2}$ no real sol

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(c) $1-x^2-3x-4>0 \Rightarrow -x^2-3x-3>0$

$x^2+3x+3<0 \Rightarrow x = \frac{-3 \pm \sqrt{9-12}}{}$ no real sol

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(9) (a) $(x+1)(x^2-x+2) \leq 0$

$x+1=0 \Rightarrow \boxed{x=-1}$

$x = \frac{1 \pm \sqrt{1-8}}{2}$ = not real sol

$\Rightarrow x^2-x+2 \geq 0$

		−1	
$x+1$	−	0	+
x^2-x+2	+	+	+
	−	0	+

why? x^2

$x^2 - 2 \cdot x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 + 2 \cdot \frac{1}{2}$

$\underbrace{\left(x-\frac{1}{2}\right)^2}_{\text{always +ive}} + \underbrace{\frac{3}{2}}_{+ive}$ } +ive

$\boxed{x \leq -1}$

(b) $2x^3-4x^2-3x>0$

$x(2x^2-4x-3)>0$

$\boxed{x=0}$ $\quad x = \frac{4 \pm \sqrt{16-4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{4 \pm \sqrt{40}}{4} = \frac{2 \pm \sqrt{10}}{2}$

$x = 1 + \frac{\sqrt{10}}{2}, \quad 1 - \frac{\sqrt{10}}{2}$

		$1-\frac{\sqrt{10}}{2}$		0		$1+\sqrt{10}/2$	
x	−	−	−	0	+	+	+
$(x-1-\frac{\sqrt{10}}{2})$	−	−	−	−	−	0	+
$(x-1+\frac{\sqrt{10}}{2})$	−	0	+	+	+	+	+
	−	0	+	0	−	0	+

$\Rightarrow \left(1-\frac{\sqrt{10}}{2}, 0\right) \cup \left(1+\frac{\sqrt{10}}{2}, \infty\right)$

(c) $x^4 + 4x^2 - 5 > 0$

$(x^2+5)(x^2-1) > 0$

always +ive

$x^2 - 1 > 0$

$\Rightarrow (x+1)(x-1) > 0$

	$-\infty$		-1		$+1$	
$(x+1)$	$-$		0	$+$	$+$	$+$
$(x-1)$	$-$		$-$	$-$	0	$+$
		$+$	0	$-$	0	$+$

$\Rightarrow (x < -1) \cup (x > 1)$

$(-\infty, -1) \cup (1, \infty) = \mathbb{R} - [-1, 1]$

(d) $16x^4 - 81 < 0$

$(4x^2 - 9)(4x^2 + 9) < 0$

$+ive$

$(2x+3)(2x-3) < 0$

	$-\infty$		$-3/2$		$3/2$	
$2x+3$	$-$		0	$+$	$+$	$+$
$2x-3$	$-$		$-$	$-$	0	$+$
		$+$	0	$-$	0	$+$

$(-3/2, 3/2)$

(10) (a) $x^2 - 2x \leq 3 \Rightarrow x^2 - 2x - 3 \leq 0$

$(x-3)(x+1) \leq 0$ solve it

(b) $1 - 3x^2 > 4 \Rightarrow -3x^2 - 3 > 0 \Rightarrow -3(x^2+1) > 0$

$\Rightarrow x^2 + 1 < 0$ no solution ϕ

$+ive$

(c) $1 < x^2 - 2x \leq 3 \Rightarrow 1 < x^2 - 2x \cap x^2 - 2x \leq 3$

$1 - x^2 + 2x < 0 \cap x^2 - 2x - 3 \leq 0$

$-x^2 + 2x + 1 < 0 \cap (x+1)(x-3) \leq 0$

$(x^2 - 2x - 1) > 0 \cap (x+1)(x-3) \leq 0$

$x = \frac{2 \pm \sqrt{4 - 4(-1)}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$x + 1 = 0 \Rightarrow x = -1$

$x - 3 = 0 \Rightarrow x = 3$

	$-\infty$		$1-\sqrt{2}$		$1+\sqrt{2}$	
$(x-1-\sqrt{2})$	$-$		$-$	$-$	0	$+$
$(x-1+\sqrt{2})$	$-$		0	$+$	$+$	$+$
		$+$	0	$-$	0	$+$

	$-\infty$		-1		3	
$(x+1)$	$-$		0	$+$	$+$	$+$
$(x-3)$	$-$		$-$	$-$	0	$+$
		$+$	0	$-$	0	$+$

$[-1, 3]$

$(-\infty, 1-\sqrt{2}) \cup (1+\sqrt{2}, \infty)$

$-1 \quad 1-\sqrt{2} \quad 1+\sqrt{2} \quad 3$

$\Rightarrow [-1, 1-\sqrt{2}) \cup (1+\sqrt{2}, 3]$