Quiz #5

Q1: $|w - 148| < 2$, w: wrestler weight.

Q2. (a) $3|x+1| - 2 = -11 \Rightarrow 3|x+1| = -9 \Rightarrow |x+1| = -3$

$S = \phi$

(b) $\frac{3x-1}{3} - \frac{2x}{x-1} = x \Rightarrow \frac{(3x-1)(x-1) - 6x}{3(x-1)} = x$

$\frac{3x^2 - 10x + 1}{3(x-1)} = x \Rightarrow 3x^2 - 10x + 1 = 3x(x-1)$

$\Rightarrow 3x^2 - 10x + 1 = 3x^2 - 3x \Rightarrow -7x = -1 \Rightarrow x = \frac{1}{7}$

$S = \{1/7\}$

(c) $\sqrt[5]{2x^2+1} - 2 = 0 \Rightarrow \sqrt[5]{2x^2+1} = 2 \Rightarrow 2x^2 + 1 = 32$

$2x^2 = 31 \Rightarrow x^2 = \frac{31}{2} \Rightarrow x = \pm \sqrt{31/2}$

$S = \{-\sqrt{\frac{31}{2}}, \sqrt{\frac{31}{2}}\}$

(d) $3x^{2/3} + 4x^{1/3} - 4 = 0 \Rightarrow (3x^{1/3} - 2)(x^{1/3} + 2) = 0$

$\Rightarrow 3x^{1/3} = 2$ or $x^{1/3} = -2 \Rightarrow x = 8/27$ or $x = -8$

$S = \{-8, \frac{8}{27}\}$

Q3 $c = \sqrt{4h(2R-h)} \Rightarrow c^2 = 4h(2R-h)$

$\Rightarrow c^2 = 8Rh - 4h^2 \Rightarrow 4h^2 - 8Rh + c^2 = 0$

$\Delta = 64R^2 - 16c^2 = 16(4R^2 - c^2) > 0$

$h = \frac{8R \pm 4\sqrt{4R^2 - c^2}}{8} \Rightarrow h = R \pm \frac{1}{2}\sqrt{4R^2 - c^2}$

Q4: ④ $x < |x-2| \leq 2$

(i) $x < |x-2|$ AND (ii) $|x-2| \leq 2$

$x < |x-2|$

$\Rightarrow x - |x-2| < 0$

x		2	
$\|x-2\|$	$-x+2$	0	$x-2$
$x-\|x-2\|<0$	$x+x-2<0$		$x-x+2<0$

- $D_1 = (-\infty, 2)$

$2x < 2$

$x < 1$

$S_1 = (-\infty, 1)$

- $D_2 = (2, \infty)$

$2 < 0$ impossible.

$S_2 = \emptyset$

$S^{(i)} = S_1 \cup S_2 = (-\infty, 1)$

$|x-2| \leq 2$

$-2 \leq x-2 \leq 2$

$0 \leq x \leq 4$

$S^{(ii)} = [0, 4]$

$S = S^{(i)} \cap S^{(ii)} = [0, 1)$

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⑤ $\frac{3}{|5-2x|} < 2 \Rightarrow |5-2x| > 3/2$

$5-2x < -3/2$ or $5-2x > 3/2$

$-2x < -\frac{13}{2}$ $-2x > \frac{3}{2} - 5$

$x > \frac{13}{4}$ $-2x > -\frac{7}{2}$

$x < 7/4$

$S = (-\infty, \frac{7}{4}) \cup (\frac{13}{4}, \infty)$

(c) $\frac{1}{(x-2)^2} > 0$ $\quad (x-2)^2$ is always positive, $x \neq 2$

$$S = (-\infty, \infty) \setminus \{2\}$$

(d) $\frac{(2x+1)^2(x-1)}{x(x^2-1)} \geq 0 \Rightarrow \frac{(2x+1)^2(x-1)}{x(x-1)(x+1)} \geq 0$

$$\Rightarrow \frac{(2x+1)^2}{x(x+1)} \geq 0 \; ; \; \boxed{x \neq 1}$$

$x = 0, \quad x = -1, \quad x = -1/2$

	-1		-1/2		0		1	
x	−		−		−	0 +	‖	+
$x+1$	−	0 +		+		+	‖	+
$\frac{(2x+1)^2}{x(x+1)}$	+	‖	−	0	−	+	‖	+

$(2x+1)^2$ is always positive, therefore it has no influence on the final result.

$$S = (-\infty, -1) \cup (0, 1) \cup (1, \infty) \cup \{-\tfrac{1}{2}\}$$