Short Papers

Dielectric Constant Measurements of Finite-Size Sheet at Microwave Frequencies by Pseudo-Brewster's Angle Method

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***Abstract*—Dielectric constant (ϵ_r) measurements for finite-size dielectric sheets (DS's) at centimeter wavelengths are presented using the method-of-measuring pseudo-Brewster's angle. This method is applied to measure ϵ_r of sheets of Plexiglas and window glass. In the experimentation, two horn antennas are used to transmit and receive p-polarized waves. A dielectric sheet is located between the two antennas and rotated 180°, which produces two peaks in the transmittance curves. For a more accurate measurement of Brewster's angle, an optical spectrometer with 1-min accuracy is also used. By this method, ϵ_r of Plexiglas and window glass are obtained to be 2.55±0.13 and 5.35±0.1, respectively. This method measures $|\epsilon_r|$, but the measurement is easy and nondestructive for DS's. Finally, an accurate method of error calculation is used to calculate the error in the measured values of ϵ_r.**

***Index Terms*—Dielectric sheet, $|\epsilon_r|$ measurement, error calculations, pseudo-Brewster's angle.**

I. Introduction

Widespread use of dielectric sheets (DS's) for microwave integrated circuits (MIC's) [1] and printed antennas, as well as their wide use in protecting [2] or improving the performance of microstrip antennas [3], has increased the demand for quick and nondestructive methods of measuring dielectric constant (ϵ_r). Measurement of ϵ_r at different frequencies requires different techniques [4]. Almost all methods of measurement are either destructive or require intense calculations. One of the rare nondestructive methods is reported to be the free-space phase measurements [5].

In this paper, the method of determining ϵ_r by measuring Brewster's angle at microwave frequencies is presented. In fact, this is a free-space method of amplitude measurement. In measurements, the distance between the antennas is barely in the far-field zone (i.e., 8–20 λ_0 where λ_0 is the free-space wavelength) and no dielectric lens is used to form a plane wave. We find that this method gives reliable and reasonably accurate values of ϵ_r by using a very simple equation [6, Ch. 1] as follows:

$$\epsilon_r = \tan^2 \theta_\beta \tag{1}$$

where θ_β is the Brewster's angle. This equation is valid for both a thick and very thin homogeneous medium [6], [7]. Here, in fact, we calculate the $|\epsilon_r|$ by measuring the pseudo-Brewster's angle [8].

II. Experimentation and Results

Dielectric sheets of Plexiglas and window (ordinary) glass of different sizes were utilized for these measurements. To observe the variation of transmitted power (T) with the angle of the incident wave, the DS's were vertically fixed on a rotating positioner. Two similar pyramidal horn antennas with 15-dB gain were placed face-to-face to each other in one line at equal distances from the DS in the H-plane (p or vertical polarization). The measurement is carried on from 7 to 13 GHz, where the accuracy of the signal frequency was 1 Hz and amplitude resolution was 0.01 dB per division on the monitor. The sweep time of the MTS-6200 was equal to the time for the positioner to rotate the DS 180°. Therefore, the x-axis of the curves, shown in Fig. 1, is the angle of the incident wave with respect to the unit vector normal at the plane of the DS, and the y-axis is the transmitted power. The detected data were saved in the MTS-6200 amplitude analyzer. To neglect the error in the starting angle due to inaccuracy in synchronization, the angle β was measured, as shown in Fig. 1, from the curves, which result in $\theta_\beta = 90° - \beta/2$. To reduce the effects of edge diffraction, the distance between the antennas and the DS was kept such that the maximum power incident on the edges of the DS was less than −10 dB. In Fig. 1, the curve b is the same as curve a—but having 2% smoothness.

With this method, an idea of the total behavior of T versus θ_1 can be observed; this is especially useful because the DS has some thickness, which produces additional changes in the curve near 90° compared to the theoretical curve. At exactly 90°, in most of the cases, an unexpected peak is observed. The measured θ_β values from the traces are not accurate enough, due to the nearly 1° error in measurement. To reduce this error, an optical spectrometer with 1-min accuracy was used. The observed data presented in Table I for the θ_β measurement using the spectrometer show much better repeatability.

III. Theoretical Method

A. Transmittance Calculation

The thickness of the DS is much smaller than λ_0; therefore, this case can be treated like thin films in optics by including the effect of multiple-beam interference [6, Ch. 7] in the DS as follows:

$$T_p = |t_p|^2 = \left[\frac{2Z}{2Z\cos\emptyset + j(Z^2+1)\sin\emptyset}\right]^2 \tag{2}$$

where

$$\emptyset = \frac{2\pi d}{\lambda_0}\sqrt{(\epsilon_r - \sin^2\theta_1)} \tag{3a}$$

$$Z = \frac{\sqrt{(\epsilon_r - \sin^2\theta_1)}}{\epsilon_r \cos\theta_1} \tag{3b}$$

and where T_p is the p-polarized transmittance, t_p is the p-polarized transmission coefficient, d is the thickness of DS, and θ_1 is the angle of incidence. The theoretical curve c, shown in Fig. 1, was obtained from calculations using (2). In the curve c, there is a clear sharp dip at 90°, while in the experimental curve a, a sharp peak is observed at 90°. This is probably due to the thickness of the DS, which, like a planar dielectric waveguide, guides the wave toward the receiving antenna. The peaks of T_p at θ_β for glass are sharper and, therefore, θ_β can be measured more accurately, while a low-ϵ_r sheet may give a flat region near θ_β. The measured values of ϵ_r given in Table I are $\epsilon_r = 2.55 \pm 0.13$ for Plexiglas and $\epsilon_r = 5.35 \pm 0.1$ for window glass. To get sharper peaks at θ_β by maximum reflection at other angles, the thickness of DS should be chosen around $d \simeq (2n+1)\lambda_0/4\sqrt{\epsilon_r}$ (where $n = 0, 1, 2, \cdots$).

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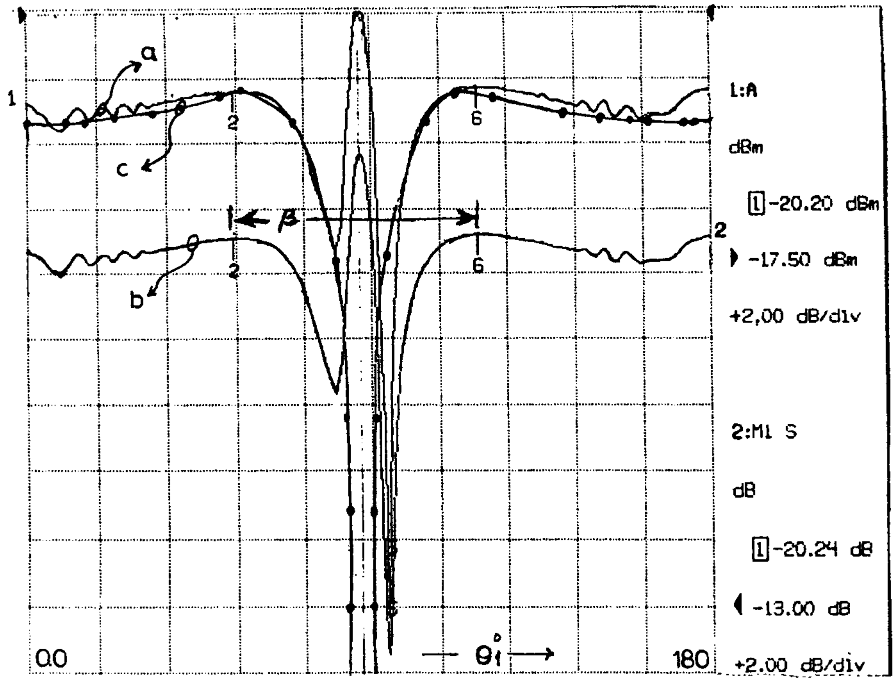

Fig. 1. The transmittance curves for window glass at 8 GHz. a—experimentally measured. b—the same curve a, but smoothed by 2%. c—theoretically calculated for comparison using (2).

TABLE I
ϵ_r DERIVED FROM MEASUREMENTS OF USING A SPECTROMETER

Dielectric Sheet Material	Dielectric Sheet Size (WxLxd) (mm)	Frequency Of Operation (GHz)	Measured Brewster's angle (degree)	Calculated Dielectric Constant (ϵ_r)
Plexiglas	480x380x3	10	58.070	2.575
	"	12	57.890	2.539
	"	12	57.233	2.414
	"	12	57.883	2.538
	"	12	58.016	2.564
	"	13	58.400	2.642
	"	13	58.050	2.571
	380x480x3	10	58.021	2.565
	"	10	57.858	2.533
	Mean Value		57.935	2.549
Window Glass	480x390x3	13	66.6666	5.374
	"	13	66.4335	5.256
	"	13	66.7666	5.426
	"	13	66.6333	5.357
	Mean Value		66.6251	5.353

B. Error Calculation

The error in measuring θ_β depends on the sensitivity and accuracy of the instruments. To calculate the error in the measurement, an accurate method of error calculation is used, which is obtained by deriving a quadratic equation from the ratio of horizontal-to-vertical transmission coefficients t_s and t_p [9]. To calculate the error in the θ_β measurement, the equations for t_s and t_p of an infinitely thick dielectric medium are used for a thin layer, knowing that the behavior of t_s and t_p is similar for thin and thick DS at Brewster's angle [6], [7]. Using equations for t_s and t_p [see (2)] of thin films in a similar manner is tedious with no advantage at θ_β.

In the case where the wave is incident from the air to the DS, the equations for t_s and t_p are [6, ch. 1]

$$t_s = \frac{2\cos\theta_1}{\cos\theta_1 + \sqrt{\epsilon_r}\cos\theta_2} \tag{4}$$

and

$$t_p = \frac{2\cos\theta_1}{\cos\theta_2 + \sqrt{\epsilon_r}\cos\theta_1} \tag{5}$$

where θ_1 is the angle of the incident wave in air and θ_2 is the similar angle in the medium. Now define a ratio named transmission coefficient ratio (TCR), which is expressed as

$$\mathrm{TCR} = \frac{t_s}{t_p} = \frac{\sqrt{\epsilon_r - \sin^2\theta_1} + \epsilon_r\cos\theta_1}{\sqrt{\epsilon_r}\cos\theta_1 + \sqrt{\epsilon_r}\sqrt{\epsilon_r - \sin^2\theta_1}}. \tag{6}$$

Substituting $U = \sin^2\theta_1$ and simplifying the above equation, we get

$$\left\{[(\epsilon_r+1)^2 - A^2]U^2 + (\epsilon_r+1)(A^2 - 4\epsilon_r)U + 4\epsilon_r^2 - A^2\epsilon_r\right\} = 0 \tag{7}$$

where

$$A = \frac{2\sqrt{\epsilon_r}(B+1)}{B-1} \tag{8a}$$

and

$$B = \left[\frac{(1-\mathrm{TCR})(1+\sqrt{\epsilon_r})}{(1+\mathrm{TCR})(1-\sqrt{\epsilon_r})}\right]^2. \tag{8b}$$

On further simplification, (7) reduces to the quadratic equation

$$(1-K)U^2 - (1+\epsilon_r)U + \epsilon_r = 0 \tag{9}$$

where

$$K = \left[\frac{(\mathrm{TCR} - \sqrt{\epsilon_r})(1 - \mathrm{TCR}\sqrt{\epsilon_r})}{\sqrt{\epsilon_r}(1-\mathrm{TCR}^2)}\right]^2. \tag{10}$$

These equations are derived for an infinitely thick DS, which, for a very thin DS, one should substitute θ_β in place of θ_1 in (6), using (9) and (10) to obtain two values for U, in which, out of the two, only the smaller value is acceptable.

The behavior of the logarithm of (10) was computed and plotted in Fig. 2 to show the variation of $\log(K)$ and TCR^2 with θ_1 from 0° to 90° for thick sheets. These results are also applicable to a thin

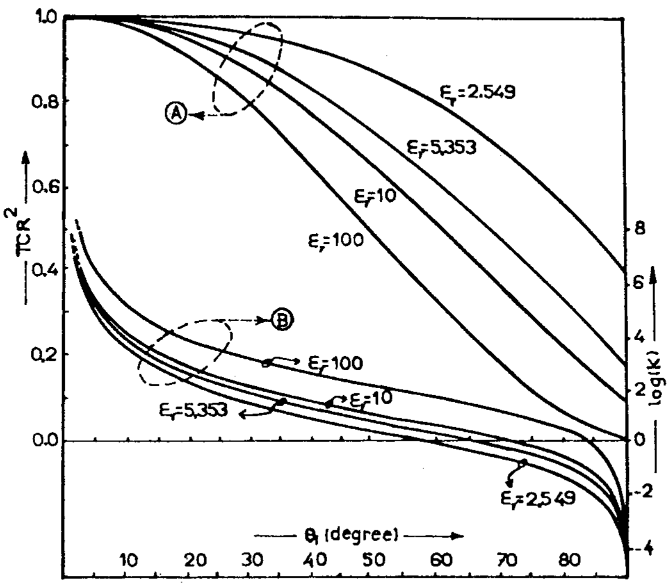

Fig. 2. Theoretical curves for different values of ϵ_r. Ⓐ—TCR^2 versus θ_1. Ⓑ—$\log(K)$ versus θ_1 (intersection of the curves with the x-axis is the relative pseudo-Brewster's angle).

TABLE II
THEORETICALLY CALCULATED TCR^2 AND PSEUDO-BREWSTER'S ANGLE FOR SPECIFIED VALUES OF S [BY (13)] AND RELATED ERRORS*

Dielectric Sheet Material	Error in Transmittance (S)	Transmittance Ratio TCR^2(at θ_1)	Incident Angle $(\theta_1)^\circ$	Error In θ $(\delta\theta_\beta)^\circ$	Error In ϵ_r $(\delta\epsilon_r)$
Plexiglas	0.0	0.89984965	57.935000	0.00	0.00
	10^{-7}	0.89984956	57.935043	9.30×10^{-5}	1.20×10^{-3}
	10^{-5}	0.89984065	57.936949	1.95×10^{-3}	1.56×10^{-3}
	10^{-3}	0.89894980	58.129928	1.95×10^{-1}	4.01×10^{-2}
	10^{-2}	0.89085115	60.896957	2.960	6.80×10^{-1}
Window glass	0.0	0.53051924	66.625095	0.00	0.00
	10^{-7}	0.53051984	66.625007	3.74×10^{-5}	9.93×10^{-5}
	10^{-5}	0.53051358	66.625458	4.13×10^{-4}	1.32×10^{-4}
	10^{-4}	0.53046566	66.628830	3.78×10^{-3}	1.86×10^{-3}
	10^{-3}	0.52998822	66.662529	3.75×10^{-2}	1.92×10^{-2}
	10^{-2}	0.52521336	66.998856	3.73×10^{-1}	1.96×10^{-1}
	10^{-1}	0.47746178	70.303833	3.6790	2.450

* Ref. value for ϵ_r of Plexiglas is 2.549 & for glass is 5.353 from Table I.

layer only near θ_β. For $\log(K)$ versus θ_1, a transition from positive to negative $\log(K)$ at θ_β is observable. Near θ_β, the theoretically calculated TCR^2 and U can be used to calculate the error in measured θ_β and, therefore, the error in ϵ_r.

In Table II, at angles near θ_β, the first values of θ_β for Plexiglas and window glass are obtained using the mean values in Table I as reference values, with the error in ϵ_r and θ_β in Table II obtained as follows:

$$|\delta\epsilon_r| = \epsilon_r(\theta_\beta^+) - \epsilon_r(\theta_\beta^-) \tag{11}$$

and

$$|\delta\theta_\beta| = \theta_\beta^+ - \theta_\beta^- \tag{12}$$

where θ_β^+ and θ_β^- are the angles measured slightly after and before the θ_β and $\epsilon_r(\theta_\beta^+)$ and $\epsilon_r(\theta_\beta^-)$ are the calculated values of ϵ_r for the corresponding θ_β, respectively. The error in measuring the transmittance ratio is calculated by using

$$|S| = \frac{\mathrm{TCR}^2(\theta_\beta^+) - \mathrm{TCR}^2(\theta_\beta^-)}{\mathrm{TCR}^2(\theta_\beta)}. \tag{13}$$

The quantity S is a specified maximum error limit, which can occur in angle measurements. For a specified value of S in Table II, the values of TCR^2 and then $\delta\epsilon_r$ and $\delta\theta_\beta$ are calculated by computer programming using (6) and (9)–(12). From the $\delta\epsilon_r$ values given in Table II, it is quite clear that for materials with higher ϵ_r, the error in ϵ_r is smaller. This has already been noted in Table I, and has also been discussed. As a typical example, for the glass DS, take 1-min error in the θ_β measurement, substitute θ_β in place of θ_1 in (6) and $\theta_\beta \pm 1$ ($\pm$1-min) in the same equation to get $\mathrm{TCR}^2(\theta_\beta^+)$ and $\mathrm{TCR}^2(\theta_\beta^-)$, respectively. Then, by utilizing (13), we obtain $|S| = 4.5\times10^{-4}$ and from (11), $\delta\epsilon_r \simeq 1.7\times10^{-2}$. The resulting calculated error in ϵ_r is almost one-sixth of the experimentally observed error, but on adding $\pm$0.1% error due to amplitude measurements to TCR^2 in (10), exactly the same error as the observed value, i.e., $\delta\epsilon_r \simeq \pm0.1$, is obtained.

IV. CONCLUSION

This paper has demonstrated that the optical method of measuring the pseudo-Brewster's angle for low-loss materials can be used for thin DS's at microwave frequencies. This method is nondestructive and the calculations involve a very simple formula. Experimental results show that it is neither necessary for the dielectric sheet to be in the far-field zone, nor is it necessary for the incident wave to be plane. However, in the far field with a planar incident wave, better results can be expected. The experimentally obtained values for the ϵ_r of Plexiglas and window glass are reliable. For dielectrics with higher values of ϵ_r, the error in measurement is reduced. Finally, an accurate method for error calculations is presented, which is obtained from a quadratic equation derived from a ratio of horizontal-to-vertical transmission coefficients. These theoretical results are confirmed by experimental observations.

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