**Assignment**

The objective is to develop a program which will calculate interpolated values of a function using Newton's method. The word "function" here is being used in a mathematical sense, and not the computer programming sense. Your program should read data from a comma separated file called "data.dat." The data will be pairs of values of x and f(x), all separated by commas, such as –2.0,5.2,-1.0,4.3

Your program will be run against such a file and it must be able to work with the data in the file and provide correct results. An explanation of interpolation, and the Newton's Method is given below:

Many problems can be defined in terms of a function. But often, this function is not known and only a small number of points is given. We must infer what we can from these points. This is done by interpolating between points. Using interpolation, we can evaluate the value of the function at any point, provided it is between the upper and lower bound of the given points. A function is generally represented as a polynomial of the form:

$p(x) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_n x^n$

where $a_0, \ldots\ldots, a_n$ are coefficients.

However, there also are other forms of polynomials, which are more relevant to interpolation, such as the Newton form:

$p(x) = a_0 + a_1(x - c_1) + a_2(x - c_1)(x - c_2) + \ldots\ldots\ldots + a_n(x - c_1)(x - c_2)\ldots\ldots(x - c_n)$

where $a_0, \ldots\ldots, a_n$ are coefficients and $c_1, \ldots\ldots, c_n$ are known as centers.

$$p_n(z) = f[x_0] + f[x_0, x_1](z - x_0) + f[x_0, x_1, x_2](z - x_0)(z - x_1) + \ldots\ldots f[x_0, x_1, \ldots, x_n](z - x_0)(z - x_1)\ldots(z - x_{n-1})$$

where $x_0,\ldots,x_n$ are the points along f(x) for which values are known, and $f[x_0]$, …, $f[x_0,\ldots,x_n]$ are divided differences, which are derived from $x_0,\ldots x_n$ and the values of f(x) at those points. Divided differences are calculated according to the formulae:

$$f[x_i] = f(x_i)$$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$$

Let's look at how you would do the calculation on paper. The first column is just the index, which always starts from zero. The second and third column values are given to us and the remaining columns are calculated.

| i | $x_i$ | $f(x_i)$ | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | -3 | -5 | $\frac{-1.1-(-5)}{-2-(-3)} = 3.9$ | $\frac{0.75-3.9}{2-(-3)} = -0.63$ | $\frac{0.43-(-0.63)}{3-(-3)} = 0.1767$ | $\frac{-0.155-0.1767}{4-(-3)} = -0.047$ | $\frac{0.027-(-0.047)}{6-(-3)} = 0.0082$ |
| 1 | -2 | -1.1 | $\frac{1.9-(-1.1)}{2-(-2)} = 0.75$ | $\frac{2.9-0.75}{3-(-2)} = 0.43$ | $\frac{-0.5-0.43}{4-(-2)} = -0.155$ | $\frac{0.058-(-0.155)}{6-(-2)} = 0.027$ | |
| 2 | 2 | 1.9 | $\frac{4.8-1.9}{3-2} = 2.9$ | $\frac{1.9-2.9}{4-2} = -0.5$ | $\frac{-0.267-(-0.5)}{6-2} = 0.058$ | | |
| 3 | 3 | 4.8 | $\frac{6.7-4.8}{4-3} = 1.9$ | $\frac{1.1-1.9}{6-3} = -0.267$ | | | |
| 4 | 4 | 6.7 | $\frac{8.9-6.7}{6-4} = 1.1$ | | | | |
| 5 | 6 | 8.9 | | | | | |

The results from the first row will be used in the calculation. The degree of the polynomial will be one less than the number of points that we had. Since in this example, we had six points, we will have a polynomial of degree five. The polynomial will be:

$$p(z) = -5 + 3.9(z+3) + (-0.63)(z+3)(z+2) + 0.1767(z+3)(z+2)(z-2) - 0.047(z+3)(z+2)(z-2)(z-3) + 0.0125(z+3)(z+2)(z-2)(z-3)(z-4)$$

Using this polynomial, we can find the value of the function at any point between –3 and 6.