Q4) $Y_1 \sim N(1,2)$, $Y_2 \sim N(2,4)$, $Y_3 \sim N(3,9)$

(5) $U = Y_1 + 2Y_2 + 3Y_3$

$M_U(t) = E e^{tU} = E e^{tY_1 + 2tY_2 + 3tY_3}$

$= M_{Y_1}(t)\, M_{Y_2}(2t)\, M_{Y_3}(3t)$

$M_X(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$

$M_U(t) = e^{t + \frac{1}{2}t^2} \cdot e^{2(2t) + \frac{4}{2}(2t)^2} \cdot e^{3(3t) + \frac{9}{2}(3t)^2}$

$= e^{t + \frac{1}{2}t^2 + 4t + \frac{16}{2}t^2 + 9t + \frac{81}{2}t^2}$

$= e^{14t + \frac{98}{2}t^2}$ MGF of $N(14, 98)$

OR) $M_U(t) = E e^{tU} = E e^{t\sum i Y_i} = \prod E e^{itY_i} = \prod M_{Y_i}(it)$

$= \prod e^{i(it) + \frac{i^2}{2}(it)^2} = \prod e^{i^2 t + \frac{i^4}{2}t^2}$

$= e^{t\sum i^2 + \frac{t^2}{2}\sum i^4}$ MGF $N(\sum i^2, \sum i^4)$

Q5) (10) $f(y_1, y_2) = e^{-(y_1+y_2)}$, $y_1 > 0$, $y_2 > 0$

$W = Y_1/(Y_1+Y_2) \Rightarrow Y_2 = \left(\frac{1-w}{w}\right) Y_1 = (1/w - 1) Y_1$

$0 < w < 1$

$\left|\frac{dy_2}{dw}\right| = \frac{y_1}{w^2}$, (2)

$g(y_1, w) = \frac{y_1}{w^2} e^{-y_1/w}$, $y_1 > 0$, $0 < w < 1$ (3)

$g(w) = \int_0^\infty \frac{y_1}{w^2} e^{-y_1/w}\, dw$

$\frac{dy_1}{dw} = -w e^{-y_1/w}$

$= \frac{1}{w^2}\left[-w y_1 e^{-y_1/w}\Big|_0^\infty + \int_0^\infty w\, e^{-y_1/w}\, dy_1\right]$

$= \frac{1}{w^2}\left[-w^2 e^{-y_1/w}\right]_0^\infty = 1$, $0 < w < 1$

$\therefore w \sim U(0, 1)$