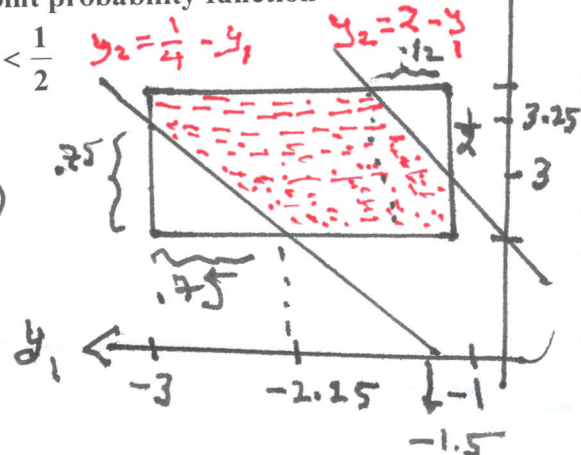


$$1 - \frac{1}{2} \left\{ \frac{1}{2} \frac{1}{2} \frac{1}{2} + \frac{1}{2} \frac{3}{4} \frac{3}{4} \right\} = 1 - \frac{10}{64} = \frac{54}{64}$$

Q3. (7 pts) Let Y_1 and Y_2 be two random variables with the following joint probability function

$$f(y_1, y_2) = \begin{cases} \frac{1}{2}, & |y_1 + 2| < 1, |y_2 - 3| < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$



1) Find $P(\frac{1}{4} < Y_1 + Y_2 < 2) = P(\frac{1}{4} - Y_1 < Y_2 < 2 - Y_1)$

$$-3 < y_1 < -1, \quad 2.5 < y_2 < 3.5$$

$$3.5 < 2 - y_2 < 3.25 \quad 3.25 < 2 - y_2$$

Sol 1: $\int_{3.25}^{3.5} \int_{-3}^{-1} \frac{1}{2} dy_1 dy_2 + \int_{\frac{1}{4} - y_2}^{-1} \int_{-2.25}^{3.5} \frac{1}{2} dy_1 dy_2$

$$+ \int_{2.5}^{3.5} \int_{\frac{1}{4} - y_2}^{-1} \frac{1}{2} dy_1 dy_2 = 0.203125 + 0.21875 + 0.375 = 0.796875$$

Sol 2: $\int_{-3}^{-2.25} \int_{\frac{1}{4} - y_1}^{3.5} \frac{1}{2} dy_2 dy_1 + \int_{-2.25}^{-1.5} \int_{2.5}^{3.5} \frac{1}{2} dy_2 dy_1 + \int_{-1.5}^{-1} \int_{2.5}^{2 - y_1} \frac{1}{2} dy_2 dy_1$

$$= 0.234375 + 0.375 + 0.1875$$

Sol 3 Comp: $1 - \left[\int_{-1.5}^{-1} \int_{2 - y_1}^{3.5} \frac{1}{2} dy_2 dy_1 + \int_{-3}^{-2.25} \int_{\frac{1}{4} - y_1}^{3.5} \frac{1}{2} dy_2 dy_1 \right]$

2) Obtain the marginal pdf of Y_1 .

$$= 1 - [0.0625 + 0.140625] = 1 - 0.203125$$

OR: $1 - \left[\int_{-3}^{-1} \int_{2 - y_2}^{-1} \frac{1}{2} dy_1 dy_2 + \int_{2.5}^{3.25} \int_{-3}^{\frac{1}{4} - y_2} \frac{1}{2} dy_1 dy_2 \right]$

$$f(y_1) = \int_{2.5}^{3.5} \frac{1}{2} dy_2 = \frac{1}{2}, \quad -3 < y_1 < -1$$