Yarmouk University	St211	6th of Jan., 09
Statistics Dept.	Final Exam	Two Hours Exam

1. (15points) If Z_1, Z_2 is a random sample from $N(0,1)$ and X_1, X_2 is a random sample from $N(1,1)$, and the two samples are independent. Then obtain the probability function of

   $a.\, \overline{X} + \overline{Z}$

   $b.\, \frac{1}{2}\left[(X_1 - X_2)^2 + (Z_1 - Z_2)^2 + (Z_1 + Z_2)^2\right]$

   $c.\, \frac{\sqrt{2}(Z_1+Z_2)}{\sqrt{(X_2-X_1)^2+(Z_2-Z_1)^2}}$

   $d.\, \frac{(X_2+X_1-2)^2}{(X_2-X_1)^2}$

   $e.\, \sum_{i=1}^{2}\left(Z_i - \overline{Z}\right)^2 + \sum_{i=1}^{2}\left(X_i - \overline{X}\right)^2.$

2. (5points) Let X_1 and X_2 be two independent random variables, such that $X_1 : Exp(2)$ and $X_2 : Exp(5)$. If $Y_2 = max(X_1, X_2)$, find $P(Y_2 < 10)$.

   Hint: $P(Y \le a) = F(a)$, where $F(\cdot)$ is the cdf of Y and a is a constant.

3. (10points) The joint pdf of Y_1 and Y_2 is

$$f(y_1, y_2) = \begin{cases} \frac{1}{4}y_1^2, & 0 \le y_2 \le y_1 \le 2 \\ 0, & \text{otherwise} \end{cases}$$

   Obtain the pdf of $U = Y_1 - Y_2$ by using the DF method.

4. (5points) Let Y_1, Y_2 and Y_3 are independent random variables such that $Y_i : N(i, i^2)$, $i = 1, 2, 3$. Use the moment generating function method to find the pdf of $U = \sum_{i=1}^{3} iY_i$.

5. (10points) Let Y_1 and Y_2 be two independent random variables, each has $Exp(1)$. Use the transformation method to obtain the pdf of $W = \frac{Y_1}{Y_1+Y_2}$.

6. (5points) Let

$$f(y_1, y_2) = \begin{cases} e^{-(y_1+y_2)}, & y_1 > 0,\, y_2 > 0 \\ 0, & \text{otherwise} \end{cases}.$$

   Find $P(Y_1 < Y_2 | Y_1 < 2Y_2)$.