Home Assignment 1

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Problem Statement: Show that the Lagrangian for a charged particle in an electromagnetic field which leads to the correct form of Newton's laws under Lorentz force is

$$L = \frac{1}{2}mv^2 - q\phi + q\vec{v}.\vec{A} \tag{1}$$

Solution: The Lagrangian of any system satisfies the Euler Lagrange Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x_i}}\right) - \frac{\partial L}{\partial x_i} = 0 \tag{2}$$

Newton's equation for a charged particle in an electromagnetic field is

$$m\ddot{x} = qE_x + \frac{q}{c}(v_yB_z - v_zB_y) \tag{3}$$

The $i = 1$ Lagrange equation is

$$\frac{d}{dt}(m\dot{x} + \frac{q}{c}A_x) = -q\frac{\partial \phi}{\partial x} + \frac{q}{c}(v_x\frac{\partial A_x}{\partial x} + v_y\frac{\partial A_y}{\partial x} + v_z\frac{\partial A_z}{\partial x}) \tag{4}$$

We now note that A_x is in general a function of all the space coordinates and teh time coordinate. Also we have $\nabla\phi$ = -E+$\frac{1}{c}\frac{\partial A}{\partial t}$

$$m\ddot{x}+\frac{q}{c}(\frac{\partial A_x}{\partial t}+v_x\frac{\partial A_x}{\partial x}+v_y\frac{\partial A_x}{\partial y}+v_z\frac{\partial A_x}{\partial z}) = qE_x+\frac{q}{c}(\frac{\partial A_x}{\partial t}+v_x\frac{\partial A_x}{\partial x}+v_y\frac{\partial A_y}{\partial x}+v_z\frac{\partial A_z}{\partial x}) \tag{5}$$

Now we cancel out the partial of A_x with respect to x from both the sides and also use the fact that $\nabla \times \vec{A} = \vec{B}$ to rearrange the above equation to

$$m\ddot{x} = qE_x + \frac{q}{c}(v_yB_z - v_zB_y) \tag{6}$$

which is same as what we would expect from the application of Newton's Law i.e equation 3. The same would apply to the $i = 2$ component and $i = 3$ component.