Home Assignment 2

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Problem : Use the Gram Schmidt Orthormalisation to find orthonormal kets given kets having the following properties.
$\langle e_1|e_1\rangle = \langle e_2|e_2\rangle = 2 \qquad \langle e_3|e_3\rangle = 4$
$\langle e_1|e_2\rangle = i\sqrt{2} \qquad \langle e_1|e_3\rangle = 1+i \qquad \langle e_2|e_3\rangle = 2$
using the Gram Schmidt normalization.

Solution : Let the first vector be $|w_1\rangle = \frac{|e_1\rangle}{\sqrt{2}}$ This is normalized as we have been given the modulus of $|e_1\rangle$. For the Gram Schmidt Normalisation we have

$$|w_{i+1}\rangle = |e_{i+1}\rangle - \Sigma_i^{j=1}\langle w_j|e_{i+1}\rangle|w_j\rangle \diagup \langle w_j|w_j\rangle \tag{1}$$

Using this and the fact that $|w_1\rangle$ is normailzed we have
$|w_2\rangle = |e_2\rangle - \langle w_1|e_2\rangle|w_1\rangle$ the unnormalised vector orthogonal to $|w_1\rangle$.
$\therefore |w_2\rangle = |e_2\rangle - \frac{i}{\sqrt{2}}|e_1\rangle$ which we find is normalised as $\langle w_2|w_2\rangle = 1$
Now we have to find out the third ket. For it we have
$|w_3\rangle = |e_3\rangle - \langle w_1|e_3\rangle|w_1\rangle - \langle w_2|e_3\rangle|w_2\rangle$
This gives
$|w_3\rangle = |e_3\rangle + b|e_1\rangle + c|w_2\rangle$
where we have the following values of a and b
$a = -(1+i)\frac{1}{2} \qquad b = \frac{1}{\sqrt{2}} - 2 - \frac{1}{\sqrt{2}i}$
Now we are left with the job of normalising these vectors. These we can do by calculating $\langle w_3|w_3\rangle$
$\langle w_3|w_3\rangle = (\langle e_3| + \frac{a^*}{\sqrt{2}}\langle e_1| + b^*\langle w_2|)(|e_3\rangle + \frac{a}{\sqrt{2}}|e_1\rangle + b|w_2\rangle)$
which afer tedious algebra reduces to
$\langle w_3|w_3\rangle = 2(\sqrt{2}-1) = c$ say
The final orthogonal ket is hence is $\frac{1}{\sqrt{c}}|w_3\rangle$