Software Packages in Physics 0332481

Session 5:

Exploration of Blackbody Radiation

Foreword:

As the 19th century turned to the 20th, a major revolution shook the world of physics. In 1900 Planck provided the basic ideas that led to the formulation of the quantum theory. In what follows, we will be content to sketch one of the basic concepts of modern physics, namely the blackbody radiation.

■Note: In addition to reviewing and simulating the physical problem at hand, this simple session aims to teach you the basics of *Maple*, and the similarities it bears to *Mathematica.*

Blackbody Radiation

We know from observing hot solids, such as the filament in a a light bulb, that if the solid gets hot enough it will begin to emit appreciable amounts of visible light. As the temperature rises, the object first becomes red hot, then more orange, and so on, until it may even appear to be blue-white.

Experiments show that some objects emit radiant energy better than others at a given temperature, and that good emitters are also good absorbers. A body that absorbs all the radiation incident on it is called an ideal *blackbody*.

The Planck formula for blackbody radiation is:

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \tag{S.5.1}$$

where $u(\lambda, T)$ is the energy density as a function of wavelength λ at temperature T, k is Boltzmann's constant, c is the speed of light, and h is Planck's constant.

Plot this energy density for several temperatures (4000, 5000, 6000K).

Derive Wien's displacement law relating T and λ_{max}, the wavelength for which the energy density is maximal.

★ Hints to Algorithm (Using *Mathematica*'s Built-in Functions)

1. To plot u vs. λ:

a. Define the energy density as a function of λ and T.

b. To plot the energy density at several temperatures, we need the values for c, h, and k.This can be obtained from the package **Miscellaneous`PhysicalConstants`**, so you have to load this package.

c. In order to get rid of units of c, h, and k, replace them with their values in inverse units, e.g., **c->SpeedOfLight(Second/Meter)**. Also you can express u in units of 10^6 J/m^3-m and λ in units of 10^4 Å.

d. To plot the energy density at three selected temperatures on the same graph, use *Mathematica*'s **Table[]** and evaluate it *before* plotting.

e. Put your plot in a frame, taking the followings into consideration: label the plot as "Energy Density vs. Wavelength", label each axis, with units, distinguish between curves belonging to different T's using colors, and name each curve with its temperature (Use *Mathematica*'s option **Epilog**).

2. To derive Wien's displacement law:

a. Differentiate $u(\lambda, T)$ with respect to λ.

b. It is obvious that the obtained derivative is a sum of fractions.This can be reduced to a single fraction using *Mathematica*'s **Together[]**.

c. Pick out the numerator (Use *Mathematica*'s **Numerator[]**), and change the variable from λ to x such that $\lambda = \frac{ch/kx}{T}$, then find the root of the obtained equation.

d. Express Wien's displacement law in terms of λ_{max}.

★ Hints to Algorithm (Using *Mathematica*'s Add-on Standard Packages)

The basic properties of black-body radiation at a specified temperature are simply given in *Mathematica*'s package **Miscellaneous`BlackBodyRadiation`**, so load the package and then:

1. Plot the spectral distribution of radiation from a blackbody for three different temperatures using *Mathematica*'s **BlackBodyProfile[]**.

2. Instead of derivation of Wien's displacement law you can find the wavelength at which the spectrum peaks using *Mathematica*'s **PeakWavelength[]**.

★ Hints to Algorithm (Using *Maple*'s Built-in Functions)

1. To plot u vs. λ:

a. Define the energy density as a function of λ and T.

b. To plot the energy density at several temperatures, assign the values of c, h, and k.

c. To generate functions of u for different values of T, make a do loop and use *Maple*'s **subs( )**.

2. To derive Wien's displacement law:

a. Reassign T as a variable.

b. Differentiate u with respect to λ using *Maple*'s **diff( )**.

c. Equate the derivative with zero and solve the obtained equation for λ using *Maple*'s **fsolve( )**.

★ Questions

1. Are your curves of energy density versus wavelength (frequency) consistent with Wien's displacement law?

2. Does a blackbody always appear black? Explain the term "blackbody".

★ Exercises

1. At what wavelength does the human body ($35\,^{o}C$) emit its maximum temperature radiation? If it really emits radiation, how can you explain the fact that it is not appearing to glow?

2. Find the peak wavelength of the radiation emitted by the sun, which has a surface temperature of about 5700K. Comment on your result.

Final Word :

- Do not forget to give physical meaning for every point.
- Try to visualize similar ideas in modern physics.
- Mention your suggestions if available .