Software Packages in Physics 0332481

Mid Term Exam (Part 2)

Date: 4/5/2004

- ***IMPORTANT INSTRUCTIONS:***

1. Write your name(s) below.
2. Solve the TWO questions below.
3. Use "Save As" to save the notebook after you finish, by changing the name of this file "midterm" to YourName

Name(s) and Registration Number(s):

QUESTION ONE: Damped Harmonic Oscillator

The equation of motion for a damped harmonic oscillator is:

$$\frac{d^2 x}{d t^2} + \gamma \frac{d x}{d t} + \omega_0^2 x = 0$$

where $\gamma = 0.5\, s^{-1}$ is the damping coefficient, and $\omega_0 = 3\, s^{-1}$ is the natural frequency.

This second-order differential equation can be reduced to two first-order differential equations, one of them can be: $\frac{d x}{d t} = v$.

1. Write down the other first-order differential equation.
2. Using $x(0) = 1$m and $v(0) = 0$ as initial conditions, solve the obtained coupled differential equations for $x(t)$ and $v(t)$.
3. Plot the phase diagram (*i.e.*, plot $v(t)$ versus $x(t)$ with t ranging from 0 to 12 s.

- Note: Your plot should be as self-contained as possible.

QUESTION TWO: Radioactive Decay

The radioactive decay of the element thorium is given by the equation:

$$N_f = N_i\, e^{-\lambda t}$$

where $\lambda = \ln 2 / t_{\frac{1}{2}} = 1.14 \times 10^{-8}\, s^{-1}$ and N_f is the number of remaining nuclei, $N_i = 1000\,$kg is the initial number of thorium nuclei t is the time elapsed, and $t_{\frac{1}{2}}$ is the half-life of the thorium. When $t = 0$, N_f is equal to N_i and no decay occurs.

1. Calculate the amount of thorium remaining after 2×10^3s, 2×10^7s, and write the result as "The amount left

after ___s is___kg".

2. Show that as t increases the amount of thorium, N_f, decreases until it becomes zero after a very long time, theoretically ∞.

Hint: Use *Mathematica*'s **Limit[]**.

Good Luck

Solution of Problem 1:

Substituting the first equation $\frac{dx}{dt} = v$ in the original equation, we get:

$$\frac{d\,v}{d\,t^2} + \gamma\,v + \omega_0^2\,x = 0$$

which is the other first-order equation. The two equations are coupled. The solution using the given initial equations is:

```
sol = DSolve[{x'[t] == v[t],
   v'[t] == -1/2 v[t] - 3^2 x[t], x[0] == 1, v[0] == 0}, {x[t], v[t]}, t]
```

$$\left\{\left\{x[t] \to \frac{1}{143} e^{-t/4} \left(143 \,\mathrm{Cos}\left[\frac{\sqrt{143}\,t}{4}\right] + \sqrt{143}\,\mathrm{Sin}\left[\frac{\sqrt{143}\,t}{4}\right]\right), v[t] \to -\frac{36\, e^{-t/4}\,\mathrm{Sin}\left[\frac{\sqrt{143}\,t}{4}\right]}{\sqrt{143}}\right\}\right\}$$

which means that the position as a function of time is given by:

```
x[t] /. sol
```

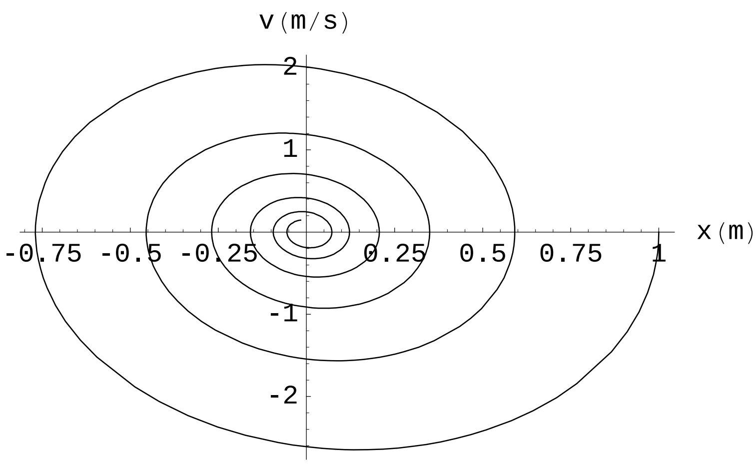
$$\left\{\frac{1}{143} e^{-t/4} \left(143 \,\mathrm{Cos}\left[\frac{\sqrt{143}\,t}{4}\right] + \sqrt{143}\,\mathrm{Sin}\left[\frac{\sqrt{143}\,t}{4}\right]\right)\right\}$$

and the velocity is:

```
v[t] /. sol
```

$$\left\{-\frac{36\, e^{-t/4}\,\mathrm{Sin}\left[\frac{\sqrt{143}\,t}{4}\right]}{\sqrt{143}}\right\}$$

```
ParametricPlot[{x[t], v[t]} /. sol,
  {t, 0, 12}, AxesLabel → {"x(m)", "v(m/s)"}];
```



Solution of Problem 2:

```
λ = 1.14 × 10^-8; Ni = 1000;

Nf[t_] := Ni Exp[-λ t]

Print["The amount left after ", 2 × 10^3, "s is ", Nf[2 × 10^3], "kg."]
Print["The amount left after ", 2 × 10^7, "s is ", Nf[2 × 10^7], "kg."]
```

```
The amount left after 2000s is 999.977kg.

The amount left after 20000000s is 796.124kg.
```

```
Limit[Nf[t], t → ∞]
```

```
0.
```

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QUESTION ONE: RLC Circuit

A circuit containing a resistor, an inductor, and a capacitor connected in series is called an RLC circuit.Using Kirchhoff's rules,one can obtain the charge on the capacitor Q and the current I as functions of time via the differential equation:

$$\frac{d^2 Q}{d t^2} + \frac{R}{L} \frac{d Q}{d t} + \frac{1}{L C} Q = 0$$

where $R = 10\,\Omega$ is the resistor, and $L = 0.1$ H is the inductance, and $C = 1 \times 10^{-6}$ F is the capacitance.

This second-order differential equation can be reduced to two first-order differential equations, one of them is: $\frac{d Q}{d t} = I$.
1. Write down the other first-order differential equation.
2. Using $Q(0) = 1$ C and $I(0) = 1$ A as initial conditions, solve the obtained coupled differential equations for $Q(t)$ and $I(t)$.
3. Plot $I(t)$ versus $Q(t)$ with t ranging from 0 to 0.03 s.
- Note: Your plot should be as self-contained as possible.

QUESTION TWO: Temperature Coefficient of Resistivity

The resistivity of a metal varies with temperature according to the expression:

$$\rho = \rho_0 \, e^{\alpha (T - T_0)}$$

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and α is the temperature coefficient of resistivity.

1. Calculate the resistivity of tungsten ($\alpha = 4.5 \times 10^{-3}$ $(°C)^{-1}$, $\rho_0 = 5.6 \times 10^{-8}$ $\Omega \cdot m$) at 100°C, and then at 120°C, then write the result as "The resistivity of tungsten at ___°C is___$\Omega \cdot$ m".
2. Show that the resistivity is given approximately by the expression $\rho = \rho_0[1 + \alpha(T - T_0)]$ for $\alpha(T - T_0) \ll 1$.

💡 Hint: Use *Mathematica*'s **`Series[]`**.

Good Luck

Solution of Problem 1:

Substituting the first equation $\frac{dQ}{dt} = I$ in the original equation, we get:

$$\frac{dI}{dt} + \frac{R}{L} I + \frac{1}{L\,C} Q = 0$$

which is the other first-order equation. The two equations are coupled. The solution using the given initial equations is:

```
sol = DSolve[{Q'[t] == i[t],
   i'[t] == -100 i[t] - 1/10^-7 Q[t], Q[0] == 1, i[0] == 1}, {Q[t], i[t]}, t]
```

$$\left\{\left\{Q[t] \to \frac{e^{-50t}\left(66650 \text{Cos}[50\sqrt{3999}\,t] + 17\sqrt{3999}\,\text{Sin}[50\sqrt{3999}\,t]\right)}{66650},\ i[t] \to -\frac{1}{1333}\left(e^{-50t}\left(-1333\,\text{Cos}[50\sqrt{3999}\,t] + 66667\sqrt{3999}\,\text{Sin}[50\sqrt{3999}\,t]\right)\right)\right\}\right\}$$

which means that the charge as a function of time is given by:

```
Q[t] /. sol
```

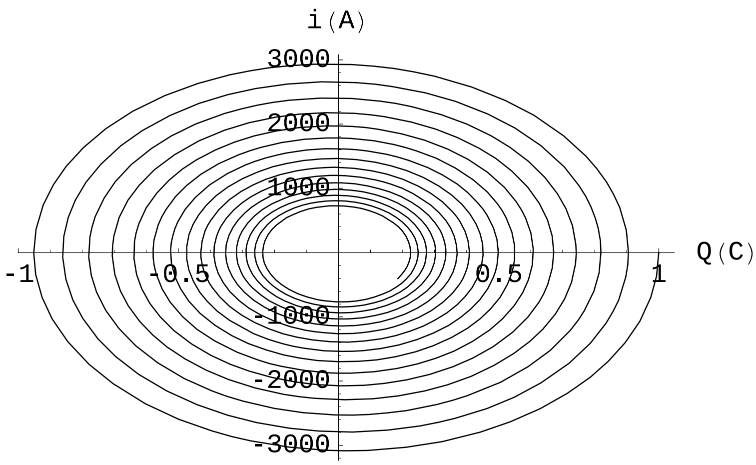
$$\left\{\frac{e^{-50t}\left(66650\,\text{Cos}[50\sqrt{3999}\,t] + 17\sqrt{3999}\,\text{Sin}[50\sqrt{3999}\,t]\right)}{66650}\right\}$$

and the current is:

```
i[t] /. sol
```

$$\left\{-\frac{e^{-50t}\left(-1333\,\text{Cos}[50\sqrt{3999}\,t] + 66667\sqrt{3999}\,\text{Sin}[50\sqrt{3999}\,t]\right)}{1333}\right\}$$

```
ParametricPlot[{Q[t], i[t]} /. sol,
  {t, 0, 0.03}, AxesLabel → {"Q(C)", "i(A)"}];
```



Solution of Problem 2:

```
α = 4.5 × 10^-3; ρ0 = 5.6 × 10^-8; T0 = 20;
```

```
ρ[T_] := ρ0 Exp[α * (T - T0)]
```

```
Print["The resistivity of tungsten at ",
 100, " °C is ", ρ[100], " Ω · m"]
Print["The resistivity of tungsten at ", 120, " °C is ", ρ[120], "Ω·m"]
```

The resistivity of tungsten at 100 °C is 8.02664×10^{-8} Ω · m

The resistivity of tungsten at 120 °C is 8.78255×10^{-8} Ω·m

```
Series[ρ0 * Exp[αα * (T - T00)], {T, T00, 1}]
```

$\rho0 + \alpha\alpha\, \rho0\, (T - T00) + O[T - T00]^2$