

$$\frac{d[\cos x]}{dx} = -\sin x$$

Proof

$$\begin{aligned}\frac{d[\cos x]}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{\cos(x + \Delta x) - \cos x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\cos x(1 - \cos \Delta x) - \sin x(\sin \Delta x)}{\Delta x} \\ &= (-\cos x) \lim_{\Delta x \rightarrow 0} \left[\frac{1 - \cos \Delta x}{\Delta x} \right] + (-\sin x) \lim_{\Delta x \rightarrow 0} \left[\frac{\sin \Delta x}{\Delta x} \right] \\ &= (-\cos x)(0) + (-\sin x)(1) = -\sin x\end{aligned}$$

Q.E.D.