$$\frac{d[e^x]}{dx} = e^x$$

Proof

Since $\ln e^x = x$

$$\frac{d[\ln e^x]}{dx} = \frac{d[x]}{dx}$$

Since $\frac{d[\ln u]}{du} = \frac{1}{u} \Rightarrow \frac{d[\ln e^x]}{dx} = \frac{d[x]}{dx} \Leftrightarrow \left(\frac{\frac{d[e^x]}{dx}}{e^x} \right) = 1$

$$\Rightarrow \frac{d[e^x]}{dx} = (e^x)(1) = e^x$$

Q.E.D.