

Theorem: **The Quotient Rule**
Proof

$$\begin{aligned}
 \frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] &= \lim_{\Delta x \rightarrow 0} \frac{\frac{f(x+\Delta x)}{g(x+\Delta x)} - \frac{f(x)}{g(x)}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{g(x)f(x+\Delta x) - f(x)g(x+\Delta x)}{\Delta x g(x)g(x+\Delta x)} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{g(x)f(x+\Delta x) - f(x)g(x) + f(x)g(x) - f(x)g(x+\Delta x)}{\Delta x g(x)g(x+\Delta x)} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{g(x)[f(x+\Delta x) - f(x)]}{\Delta x} - \frac{f(x)[g(x+\Delta x) - g(x)]}{\Delta x}}{g(x)g(x+\Delta x)} \\
 &= \frac{g(x) \left[\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \right] - f(x) \left[\lim_{\Delta x \rightarrow 0} \frac{g(x+\Delta x) - g(x)}{\Delta x} \right]}{\lim_{\Delta x \rightarrow 0} [g(x)g(x+\Delta x)]} \\
 &= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}
 \end{aligned}$$

Q.E.D.