$$\frac{d[\tan x]}{dx} = \sec^2 x$$

Proof

$$\frac{d[\tan x]}{dx} = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{(\cos x)(\cos x) - (-\sin x)(\sin x)}{[\cos x]^2} \qquad \text{(by the Quotient Rule)}$$

$$= \frac{\cos^2 x + \sin^2 x}{[\cos x]^2} = \frac{1}{[\cos x]^2} = \sec^2 x$$

Q.E.D.

$$\frac{d[\sec x]}{dx} = \sec x \tan x$$

Proof

$$\frac{d[\sec x]}{dx} = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{(\cos x)(0) - (1)(-\sin x)}{[\cos x]^2} = \frac{\sin x}{\cos^2 x} = \sec x \tan x$$

Q.E.D.

$$\frac{d[\csc x]}{dx} = -\cot x \csc x$$

Proof

$$\frac{d[\csc x]}{dx} = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (\cos x)(1)}{(\sin x)^2} = \frac{-\cos x}{\sin^2 x} = -\cot x \csc x$$

Q.E.D.

$$\frac{d[\cot x]}{dx} = -\csc^2 x$$

Proof

$$\frac{d[\cot x]}{dx} = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x} = -\csc^2 x$$

Q.E.D.