

$$\int a^x dx = \left(\frac{1}{\ln a} \right) a^x + C$$

Proof

$$\text{Since } e^{\ln a} = a \Rightarrow \int a^x dx = \int e^{(\ln a)x} dx$$

$$\text{Let } u = (\ln a)x \Rightarrow du = (\ln a)dx \Rightarrow dx = \frac{du}{\ln a}$$

$$\therefore \int a^x dx = \int e^{(\ln a)x} dx = \frac{1}{\ln a} \int e^u du = \frac{1}{\ln a} (e^u + K) = \frac{1}{\ln a} (e^{(\ln a)x} + K)$$

$$= \left(\frac{1}{\ln a} \right) a^x + C \quad . \quad \text{Note that } 1/\ln a \text{ is a constant, therefore } K/\ln a \text{ is also a constant.}$$

Q.E.D.