$$\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$$

Proof

$$\text{Since } \int \cos^n x dx = \int (\cos^{n-1} x)(\cos x) dx$$

$$\text{Let} \quad u = \cos^{n-1} x \qquad v = \sin x$$

$$du = -(n-1)\cos^{n-2} x \sin x \qquad dv = \cos x dx$$

$$\therefore \int (\cos^{n-1} x)(\cos x) dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x \sin^2 x dx$$

$$\text{Since} \quad \sin^2 x = 1 - \cos^2 x$$

$$\cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x \sin^2 x dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x(1-\cos^2 x) dx$$

$$= \cos^{n-1} x \sin x + (n-1)\left\{\int \cos^{n-2} x dx - \int \cos^n x dx\right\}$$

$$= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x dx - (n-1)\int \cos^n x dx$$

$$\text{Since} \quad \int \cos^n x dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x dx - (n-1)\int \cos^n x dx$$

$$\int \cos^n x dx + (n-1)\int \cos^n x dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x dx$$

$$\Leftrightarrow n\int \cos^n x dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x dx$$

$$\Leftrightarrow \int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x dx$$

Q.E.D