Definition of Two Special Definite Integrals

1. If f is defined at x = a, then $\int_a^a f(x)dx = 0.$

2. if f is integrable on [a, b], then $\int_b^a f(x)dx = -\int_a^b f(x)dx.$

Theorem : Additive Interval Property

If f is integrable on the three closed intervals determined by a, b, and c, then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx.$$

Theorem : Properties of Definite Integrals

If f and g are integrable on [a, b] and k is a contant, then the functions of kf and $f \pm g$ are integrable on [a, b], and

1. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$

2. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx.$

Theorem : Preservation of Inequality

1. If f is integrable and nonnegative on the closed interval [a, b], then

$$0 \le \int_a^b f(x)dx.$$

2. If f and g are integrable on the closed interval [a, b] and $f(x) \le g(x)\ \forall x \in [a,b]$, then

$$\int_a^b f(x)dx \le \int_a^b g(x)dx.$$

Theorem : The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval [a, b] and F is an antiderivative of f on the interval [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a).$$

Theorem : The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x).$$