Theorem: **Integral of a Quadratic Function**

If $p(x) = Ax^2 + Bx + C$, then

$$\int_a^b p(x)dx = \left(\frac{b-a}{6}\right)\left[p(a) + 4p\left(\frac{a+b}{2}\right) + p(b)\right].$$

Proof

$$p(x)dx = \int_a^b (Ax^2 + Bx + C)dx = \left[\frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx\right]_a^b$$

$$= \frac{A(b^3 - a^3)}{3} + \frac{B(b^2 - a^2)}{2} + C(b-a)$$

$$\int_a^b = \left(\frac{b-a}{6}\right)\left[2A(a^2 + ab + b^2) + 3B(b+a) + 6C\right]$$

$$= \left(\frac{b-a}{6}\right)\left[p(a) + 4p\left(\frac{a+b}{2}\right) + p(b)\right]$$

done