$$\int \sin^n x dx = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x dx$$

Proof

$$\int \sin^n x dx = \int (\sin^{n-1} x)(\sin x) dx$$

let $u = \sin^{n-1} x$ $\qquad v = -\cos x$

$du = (n-1)\sin^{n-2} x \cos x \qquad dv = \sin x dx$

$$\therefore \int \sin^n x dx = \sin^{n-1} x \cos x - (n-1)\int \sin^{n-2} x \cos^2 x dx$$

Since $\cos^2 x = 1 - \sin^2 x$

$$\begin{aligned}\int \sin^n x dx &= \sin^{n-1} x \cos x - (n-1)\int \sin^{n-2} x \cos^2 x dx \\ &= \sin^{n-1} x \cos x - (n-1)\int \{\sin^{n-2} x(1-\sin^2 x)\} dx \\ &= \sin^{n-1} x \cos x - (n-1)\int \sin^{n-2} x dx + (n-1)\int \sin^n x dx\end{aligned}$$

$$\therefore \int \sin^n x dx = \sin^{n-1} x \cos x - (n-1)\int \sin^{n-2} x dx + (n-1)\int \sin^n x dx$$

$$\Leftrightarrow \int \sin^n x dx - (n-1)\int \sin^n x dx = \sin^{n-1} x \cos x - (n-1)\int \sin^{n-2} x dx$$

$$\Leftrightarrow -n\int \sin^n x dx = \sin^{n-1} x \cos x - (n-1)\int \sin^{n-2} x dx$$

$$\Leftrightarrow \int \sin^n x dx = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x dx$$

Q.E.D.