If a and b are real numbers and u is differentiable then,

$$\int\sqrt{a^2 \pm b^2 - u^2} \Rightarrow u = \sqrt{a^2 \pm b^2}\sin\theta \Rightarrow du = \sqrt{a^2 \pm b^2}\cos\theta d\theta$$

$$\therefore \sqrt{a^2 \pm b^2 - u^2} = \sqrt{(a^2 \pm b^2) - (a^2 \pm b^2)\sin^2\theta} = \sqrt{(a^2 \pm b^2)(1-\sin^2\theta)} = \sqrt{(a^2 \pm b^2)\cos^2\theta}$$

$$= \sqrt{(a^2 \pm b^2)}\cos\theta$$

$$\int\sqrt{a^2 \pm b^2 - u^2} = \int(\sqrt{a^2 \pm b^2})^2\cos^2\theta d\theta = (a^2 \pm b^2)\int\cos^2\theta d\theta$$

To compute $\int\cos^2\theta d\theta$ use integration by parts: $\left(\int rdq = rq - \int qdr\right)$

Let $r = \cos\theta \Rightarrow dr = -\sin\theta d\theta$ and $dq = \cos\theta d\theta \Rightarrow q = \sin\theta$

$$\therefore \int\cos^2\theta d\theta \Leftrightarrow \cos\theta\sin\theta - \int\sin\theta(-\sin\theta d\theta) = \cos\theta\sin\theta + \int\sin^2\theta d\theta = \cos\theta\sin\theta + \int(1-\cos^2\theta d\theta)$$

$$\cos\theta\sin\theta + \int d\theta - \int\cos^2\theta d\theta \Leftrightarrow 2\int\cos^2\theta d\theta = \cos\theta\sin\theta + \theta \Leftrightarrow \int\cos^2\theta d\theta = \frac{1}{2}(\cos\theta\sin\theta + \theta)$$

$$\therefore (a^2 \pm b^2)\int\cos^2\theta d\theta = \frac{1}{2}(a^2 \pm b^2)(\cos\theta\sin\theta + \theta) + C$$

Since u= $\sqrt{a^2 \pm b^2}\sin\theta \Rightarrow \sin\theta = \frac{u}{\sqrt{a^2 \pm b^2}}$ this implies that the opposite side of the right triangle is u and that the hypotenuse is $\sqrt{a^2 \pm b^2}$. Moreover, adjacent side is $\sqrt{a^2 \pm b \ \ - u^2}$.

Therefore, $\cos\theta = \frac{\sqrt{a^2 \pm b^2 - u^2}}{\sqrt{a^2 \pm b^2}}$ and $\theta = \arcsin\frac{u}{\sqrt{a^2 \pm b^2}}$.

Thus,

$$\frac{1}{2}(a^2 \pm b^2)(\cos\theta\sin\theta + \theta) = \frac{1}{2}(a^2 \pm b^2)\left(\frac{\sqrt{a^2 \pm b^2 - u}}{\sqrt{a^2 \pm b^2}}\frac{u}{\sqrt{a^2 \pm b^2}} + \arcsin\frac{u}{\sqrt{a^2 \pm b^2}}\right)$$

$$= \frac{1}{2}\left(u\sqrt{a^2 \pm b^2 - u^2} + (a^2 \pm b^2)\arcsin\frac{u}{\sqrt{a^2 \pm b^2}}\right)$$